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magnetohydrodynamics in the Earth's core Scale disparities and

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Keke Zhang and David Gubbins

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Scale disparities and magnetohydrodynamics
in the Earth's core ties and magnetohydro
in the Earth's core in the Earth's core
 By KEKE Z HANG¹ AND DAVID GUBBINS²

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 \mathcal{F} Fluid motions driven by convection in the Earth's fluid core sustain geomagnetic
fields by magnetohydrodynamic dynamo processes. The dynamics of the core is crit-Fluid motions driven by convection in the Earth's fluid core sustain geomagnetic
fields by magnetohydrodynamic dynamo processes. The dynamics of the core is crit-
ically influenced by the combined effects of rotation and m Fluid motions driven by convection in the Earth's fluid core sustain geomagnetic
fields by magnetohydrodynamic dynamo processes. The dynamics of the core is crit-
ically influenced by the combined effects of rotation and m fields by magnetohydrodynamic dynamo processes. The dynamics of the core is critically influenced by the combined effects of rotation and magnetic fields. This paper attempts to illustrate the scale-related difficulties in ically influenced by the combined effects of rotation and magnetic fields. This paper
attempts to illustrate the scale-related difficulties in modelling a convection-driven
geodynamo by studying both linear and nonlinear c attempts to illustrate the scale-related difficulties in modelling a convection-driven
geodynamo by studying both linear and nonlinear convection in the presence of
imposed toroidal and poloidal fields. We show that there geodynamo by studying both linear and nonlinear convection in the presence of imposed toroidal and poloidal fields. We show that there exist three extremely large disparities, as a direct consequence of small viscosity and imposed toroidal and poloidal fields. We show that there exist three extremely large
disparities, as a direct consequence of small viscosity and rapid rotation of the Earth's
fluid core, in the spatial, temporal and amplit disparities, as a direct consequence of small viscosity and rapid rotation of the Earth's
fluid core, in the spatial, temporal and amplitude scales of a convection-driven geo-
dynamo. We also show that the structure and st fluid core, in the spatial, temporal and amplitude scales of a convection-driven geo-
dynamo. We also show that the structure and strength of convective motions, and,
hence, the relevant dynamo action, are extremely sensit dynamo. We also show that the structure and strength of convective motions, and, hence, the relevant dynamo action, are extremely sensitive to the intricate dynamical balance between the viscous, Coriolis and Lorentz force hence, the relevant dynamo action, are extremely sensitive to the intricate dynamical
balance between the viscous, Coriolis and Lorentz forces; similarly, the structure and
strength of the magnetic field generated by the d balance between the viscous, Coriolis and Lorentz forces; similarly, the structure and
strength of the magnetic field generated by the dynamo process can depend very
sensitively on the fluid flow. We suggest, therefore, th strength of the magnetic field generated by the dynamo process can depend very
sensitively on the fluid flow. We suggest, therefore, that the zero Ekman number
limit is strongly singular and that a stable convection-driven sensitively on the fluid flow. We suggest, therefore, that the zero Ekman number
limit is strongly singular and that a stable convection-driven strong-field geodynamo
satisfying Taylor's constraint may not exist. Instead, limit is strongly singular and that a stable convection-driven strong-field geodynamo
satisfying Taylor's constraint may not exist. Instead, the geodynamo may vacillate
between a strong field state, as at present, and a we satisfying Taylor's constraint may not exist. Instead, the geodynamo may vacillate

Keywords: geodynamo; Taylor constraint; Earth's core; magnetoconvection

1. Introduction

1. Introduction
The primary dynamics of the Earth's fluid core is controlled by (1) rapid rotation,
(2) small viscosity (3) thermal or compositional convection, and (4) a self-generated The primary dynamics of the Earth's fluid core is controlled by (1) rapid rotation,

(2) small viscosity, (3) thermal or compositional convection, and (4) a self-generated

magnetic field (Moffatt, 1978; Busse 1978; Gubbi The primary dynamics of the Earth's fluid core is controlled by (1) rapid rotation, (2) small viscosity, (3) thermal or compositional convection, and (4) a self-generated magnetic field (Moffatt 1978; Busse 1978; Gubbins \blacktriangleleft (2) small viscosity, (3) thermal or compositional convection, and (4) a self-generated magnetic field (Moffatt 1978; Busse 1978; Gubbins & Roberts 1987; Roberts & \blacktriangleright Soward 1992; Hollerbach 1996; Fearn 1997). ity, variable rotation, boundary conditions, or the origin of buoyancy (thermal or Soward 1992; Hollerbach 1996; Fearn 1997). Other details such as compressibility, variable rotation, boundary conditions, or the origin of buoyancy (thermal or compositional) are of secondary importance to the dynamics on ity, variable rotation, boundary conditions, or the origin of buoyancy (thermal or compositional) are of secondary importance to the dynamics on the long (magnetic-
diffusion) time-scale, for which the Coriolis force must forces,

$$
2\Omega \times \mathbf{u} = -\frac{1}{\rho} \nabla p + \alpha \Theta g_0 \mathbf{r} + \frac{1}{\rho \mu} (\nabla \times \mathbf{B}) \times \mathbf{B} + \nu \nabla^2 \mathbf{u},
$$
 (1.1)

 $2\Omega \times \mathbf{u} = -\frac{1}{\rho} \nabla p + \alpha \Theta g_0 \mathbf{r} + \frac{1}{\rho \mu} (\nabla \times \mathbf{B}) \times \mathbf{B} + \nu \nabla^2 \mathbf{u},$ (1.1)
where **r** is the position vector, ρ the mean density of the Earth's liquid core, g_0
the acceleration of gravity Ω the where **r** is the position vector, ρ the mean density of the Earth's liquid core, g_0 the acceleration of gravity, Ω the angular velocity of the Earth, Θ the deviation of temperature from the adiabatic ν the where **r** is the position vector, ρ the mean density of the Earth's liquid core, g_0 the acceleration of gravity, Ω the angular velocity of the Earth, Θ the deviation of temperature from the adiabatic, ν the the acceleration of gravity, Ω the angular velocity of the Earth, Θ the deviation of temperature from the adiabatic, ν the kinematic viscosity, μ the magnetic permeability, α the thermal expansion coefficie ability, α the thermal expansion coefficient, **u** the velocity field, and **B** the generated
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magnetic field. In equation (1.1), $2\Omega \times u$ is the Coriolis force, $-(\nabla p)/\rho$ the pres-**IATHEMATICAL,
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CIENCES** magnetic field. In equation (1.1), $2\Omega \times u$ is the Coriolis force, $-(\nabla p)/\rho$ the pressure force, $\alpha \Theta g_0 \mathbf{r}$ the buoyancy force, $(\nabla \times \mathbf{B}) \times \mathbf{B}/\rho\mu$ the magnetic (Lorentz) force, and $\nu \nabla^2 u$ the viscous f sure force, $\alpha \Theta q_0$ **r** the buoyancy force, $(\nabla \times \mathbf{B}) \times \mathbf{B}/\rho \mu$ the magnetic (Lorentz) sure force, $\alpha\Theta g_0$ **r** the buoyancy force, $(\nabla \times \mathbf{B}) \times \mathbf{B}/\rho\mu$ the magnetic (Lorentz)
force, and $\nu\nabla^2\mathbf{u}$ the viscous force. The inertial force $(\partial \mathbf{u}/\partial t + \mathbf{u} \cdot \nabla \mathbf{u})$ has been
neglected because i force, and $\nu\nabla^2\mathbf{u}$ the viscous force. The inertial force $(\partial \mathbf{u}/\partial t + \mathbf{u} \cdot \nabla \mathbf{u})$ has been neglected because its contribution on the magnetic diffusion time-scale is small.
On the shorter, century-long, ad neglected because its contribution on the magnetic diffusion time-scale is small.
On the shorter, century-long, advection time-scale, however, inertia may play an
important role in providing an extra way to relax the rotat important role in providing an extra way to relax the rotational constraint or for
a dynamo solution to remain close to the Taylor constraint (see equation (1.2)
below). The centrifugal force, $\rho \Omega \times (\Omega \times r)$, has been abs a dynamo solution to remain close to the Taylor constraint (see equation (1.2) below). The centrifugal force, $\rho \Omega \times (\Omega \times r)$, has been absorbed into the modified
pressure, *p*.
The small viscosity makes equation (1.1) very difficult to treat, and two different
approaches have been developed to solve

pressure, p.
The small viscosity makes equation (1.1) very difficult to treat, and two different
approaches have been developed to solve it. The first assumes that the zero viscosity
limit, $\nu \to 0$, is non-singular. Sett The small viscosity makes equation (1.1) very difficult to treat, and two different
approaches have been developed to solve it. The first assumes that the zero viscosity
limit, $\nu \to 0$, is non-singular. Setting $\nu = 0$, approaches have been developed to solve it. The first assumes that the zero viscosity
limit, $\nu \to 0$, is non-singular. Setting $\nu = 0$, integrating the $\hat{\phi}$ -component of (1.1)
over the surface of a geostrophic cylinde limit, $\nu \to 0$, is non-singular. Setting $\nu = 0$, integrating the ϕ -component of (1.1) over the surface of a geostrophic cylinder $G(s)$ with radius s and axis parallel to that of rotation, and using the incompressibil over the surface of a geostrophic cylinder $G(s)$ with radius s and axis parallel to that of rotation, and using the incompressibility condition $\nabla \cdot \mathbf{u} = 0$, gives Taylor's constraint (Taylor 1963; see also Jault 1995 1999):

$$
\int_{G(s)} [(\nabla \times \mathbf{B}) \times \mathbf{B}]_{\phi} dS = 0.
$$
\n(1.2)

The magnetic field must satisfy Taylor's constraint for (1.1) to hold without viscos-The magnetic field must satisfy Taylor's constraint for (1.1) to hold without viscos-
ity; if such a solution exists it is called a *strong-field* dynamo. There have been many
attempts to obtain a convection-driven stro The magnetic field must satisfy Taylor's constraint for (1.1) to hold without viscosity; if such a solution exists it is called a *strong-field* dynamo. There have been many attempts to obtain a convection-driven strong ity; if such a solution exists it is called a *strong-field* dynamo. There have been many
attempts to obtain a convection-driven strong-field dynamo where viscosity plays
at most a minor role in boundary layers (see, for e attempts to obtain a convection-driven
at most a minor role in boundary layers
Walker *et al.* 1998), but all have failed.
The second approach is to replace the most a minor role in boundary layers (see, for example, Fearn & Proctor 1987;
alker *et al.* 1998), but all have failed.
The second approach is to replace the small viscosity of the Earth's core by
much larger one, someti

Walker *et al.* 1998), but all have failed.
The second approach is to replace the small viscosity of the Earth's core by
a much larger one, sometimes with an artificial form of hyperviscosity in which
small length-scales The second approach is to replace the small viscosity of the Earth's core by
a much larger one, sometimes with an artificial form of hyperviscosity in which
small length-scales see a higher effective viscosity. This has r a much larger one, sometimes with an artificial form of hyperviscosity in which
small length-scales see a higher effective viscosity. This has resulted in consider-
able progress (see, for example, Glatzmaier & Roberts 19 small length-scales see a higher effective viscosity. This has resulted in considerable progress (see, for example, Glatzmaier & Roberts 1995 a, b , 1996 a, b ; Kuang & Bloxham 1997; Jones *et al.* 1995; Sarson & Jones 1999 able progress (see, for example, Glatzmaier & Roberts 1995 a, b , 1996 a, b ; Kuang & Bloxham 1997; Jones *et al.* 1995; Sarson & Jones 1999). The resulting dynamos are able to generate magnetic fields and flows of the righ Bloxham 1997; Jones *et al.* 1995; Sarson & Jones 1999). The resulting dynamos are able to generate magnetic fields and flows of the right strength and morphology to model the Earth; they also have the potential to explai are able to generate magnetic fields and flows of the right strength and morphology
to model the Earth; they also have the potential to explain geomagnetic reversals.
However, they are in the wrong regime for the Earth, wi to model the Earth; they also have the potential to explain geomagnetic reversals.
However, they are in the wrong regime for the Earth, with viscous forces playing
a significant dynamical role. This makes comparing the num However, they are in the wrong regime for the Earth, with viscous forces playing a significant dynamical role. This makes comparing the numerical simulations with observations hazardous: an Earth-like polarity reversal may a significant dynamical role. This makes comparing the numerical simulations with core.

At first glance, the strong-field dynamo presents the most attractive model of magnetohydrodynamics in the Earth's core: it has a large-scale convective flow, a At first glance, the strong-field dynamo presents the most attractive model of magnetohydrodynamics in the Earth's core: it has a large-scale convective flow, a strong toroidal magnetic field, nearly negligible viscous dis magnetohydrodynamics in the Earth's core: it has a large-scale convective flow, a
strong toroidal magnetic field, nearly negligible viscous dissipation, and an efficient
thermal dynamo engine. In practice, such dynamos fai strong toroidal magnetic field, nearly negligible viscous dissipation, and an efficient
thermal dynamo engine. In practice, such dynamos fail in a number of ways (see, for
example, Fearn 1998), most instructively, when the \bigcup thermal dynamo engine. In practice, such dynamos fail in a number of ways (see, for \bigcirc example, Fearn 1998), most instructively, when the dynamo equations are integrated \bigcirc in time starting from initial condit tion. In magnetoconvection, the field is imposed; once time integration starts, the in time starting from initial conditions provided by a magnetoconvection calculation. In magnetoconvection, the field is imposed; once time integration starts, the imposed field is removed, leaving only that generated by tion. In magnetoconvection, the field is imposed; once time integration starts, the imposed field is removed, leaving only that generated by the dynamo process. One of us (K.Z.) has conducted a number of such numerical exp imposed field is removed, leaving only that generated by the dynamo process. One of
us (K.Z.) has conducted a number of such numerical experiments with similar results.
The strength of the initial magnetic field gradually us (K.Z.) has conducted a number of such numerical experiments with similar results.
The strength of the initial magnetic field gradually decreases over a few magnetic
diffusion times and, at the same time, small-scale con inant. Dynamo action subsequently collapses completely because the amplitude of

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Scale disparities in the geodynamo 901
convection drops below the critical value; sometimes even the convection shuts off
because the Ravleigh number falls below the critical value. Increasing the viscosity, convection drops below the critical value; sometimes even the convection shuts off because the Rayleigh number falls below the critical value. Increasing the viscosity, or introducing hyperdiffusivity, prevents this collap convection drops below the critical value; sometimes even the convection shuts off because the Rayleigh number falls below the critical value. Increasing the viscosity, or introducing hyperdiffusivity, prevents this collap because the Rayleigh number falls below the critical value. Increasing the viscosity,
or introducing hyperdiffusivity, prevents this collapse because the additional viscos-
ity prevents dominance of small-scale convection or introducing hyperdiffusivity, prevents this collapse because the additional viscos-
ity prevents dominance of small-scale convection at times when the field is weak. The
artificially high viscosity is, therefore, respon ity prevents dominance of small-scale convection a
artificially high viscosity is, therefore, responsible
hardly what we imagine happening in the core.
The first aim of numerical modelling is not to

tificially high viscosity is, therefore, responsible for sustaining the dynamo action, rdly what we imagine happening in the core.
The first aim of numerical modelling is not to reproduce exactly the right val-
s of the pa hardly what we imagine happening in the core.
The first aim of numerical modelling is not to reproduce exactly the right values of the parameters in the core, but to approach the correct dynamical regime. The first aim of numerical modelling is not to reproduce exactly the right values of the parameters in the core, but to approach the correct dynamical regime.
We have so far been unable to do this for the geodynamo because ues of the parameters in the core, but to approach the correct dynamical regime.
We have so far been unable to do this for the geodynamo because of the small
viscosity, yet magnetoconvection calculations can be extrapolate We have so far been unable to do this for the geodynamo because of the small viscosity, yet magnetoconvection calculations can be extrapolated to very small viscosity quite realistically. Why is the dynamo calculation so v viscosity, yet magnetoconvection calculations can be extrapolated to very small viscosity quite realistically. Why is the dynamo calculation so very much more difficult than magnetoconvection with the same imposed paramete cosity quite realistically. Why is the dynamo calculation so very much more difficult than magnetoconvection with the same imposed parameters? We argue here that it is because the self-generated field strength can vary, le cult than magnetoconvection with the same imposed parameters? We argue here that it is because the self-generated field strength can vary, leading to huge ranges of time-, length- and amplitude scales that are very hard to that it is because the self-generated field strength can vary, leading to huge ranges
of time-, length- and amplitude scales that are very hard to deal with numer-
ically. The problems at small Ekman number are, therefore, of time-, length- and amplitude scales that are very hard to deal with numerically. The problems at small Ekman number are, therefore, much more subtle Earth. than simply resolving boundary layers or achieving exactly the right scales for the Earth.
We suggest here that the strong-field dynamo is in fact unstable and prone to

Earth.
We suggest here that the strong-field dynamo is in fact unstable and prone to
collapse into a weak-field state similar to non-magnetic convection. The argument
is based on results from magnetoconvection extrapolated We suggest here that the strong-field dynamo is in fact unstable and prone to collapse into a weak-field state similar to non-magnetic convection. The argument is based on results from magnetoconvection extrapolated to the collapse into a weak-field state similar to non-magnetic convection. The argument
is based on results from magnetoconvection extrapolated to the very small values
of viscosity found in the core; it receives some support fr is based on results from magnetoconvection extrapolated to the very small values
of viscosity found in the core; it receives some support from recent palaeomagnetic
evidence that the Earth's magnetic field collapses and al of viscosity found in the core; it receives some support from recent palaeomagnetic
evidence that the Earth's magnetic field collapses and almost reverses many times
between full polarity reversals (Gubbins 1999), and that evidence that the Earth's magnetic field collapses and almost reverses many
between full polarity reversals (Gubbins 1999), and that its amplitude varies
matically on a millennium time-scale (see, for example, Channel, thi matically on a millennium time-scale (see, for example, Channel, this issue).
2. Mathematical formulation

2. Mathematical formulation
To a first approximation, the dynamics of the Earth's liquid core is governed by the
following equations of motion, heat and induction in a spherical shell of electrically To a first approximation, the dynamics of the Earth's liquid core is governed by the following equations of motion, heat and induction in a spherical shell of electrically conducting Boussines fluid with inner radius r_i To a first approximation, the dynamics of the Earth's liquid core is got following equations of motion, heat and induction in a spherical shell conducting Boussinesq fluid with inner radius r_i and outer radius r_o : \frac

ducting Boussinesq fluid with inner radius
$$
r_1
$$
 and outer radius r_0 :
\n
$$
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + 2\Omega \mathbf{k} \times \mathbf{u} = -\frac{1}{\rho} \nabla p + \alpha \Theta g_0 \mathbf{r} + \frac{1}{\rho \mu} (\nabla \times \mathbf{B}) \times \mathbf{B} + \nu \nabla^2 \mathbf{u}, \quad (2.1)
$$

$$
\frac{\partial \Theta}{\partial t} + \mathbf{u} \cdot \nabla \Theta + \mathbf{u} \cdot \nabla T_s = \kappa \nabla^2 \Theta,
$$
\n(2.2)

$$
\frac{\partial}{\partial t} \mathbf{B} + \mathbf{u} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{u} + \lambda \nabla^2 \mathbf{B},
$$
\n(2.3)

 $\frac{\partial}{\partial t} \mathbf{B} + \mathbf{u} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{u} + \lambda \nabla^2 \mathbf{B},$ (2.3)
where **k** is a unit vector parallel to the axis of rotation, $T_s = \beta r^2/2$ is a basic unstable
temperature produced by the uniform distribution o ∂t
where **k** is a unit vector parallel to the axis of rotation, $T_s = \beta r^2/2$ is a basic unstable
temperature produced by the uniform distribution of heat sources (Chandrasekhar
1961: Roberts 1968) and t is time where **k** is a unit vector parallel to the
temperature produced by the uniform
1961; Roberts 1968), and t is time.
Equations $(2\ 1)-(2\ 3)$ can be nontemperature produced by the uniform distribution of heat sources (Chandrasekhar 1961; Roberts 1968), and t is time.

Equations (2.1) – (2.3) can be non-dimensionalized as follows:

$$
\rightarrow dr, \qquad t \rightarrow td^2/\kappa, \qquad \Theta \rightarrow \Theta \beta d^2, \qquad \mathbf{b} \rightarrow B_0 \mathbf{b}, \tag{2.4}
$$

 $r \to dr$, $t \to td^2/\kappa$, $\Theta \to \Theta \beta d^2$, $\mathbf{b} \to B_0 \mathbf{b}$, (2.4)
where $d = (r_0 - r_i)$ and B_0 is a typical amplitude of the generated magnetic field. In
dimensionless form the governing equations become where $d = (r_o - r_i)$ and B_0 is a typical amplitude of the
dimensionless form, the governing equations become

$$
EPr^{-1}\left|\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right| + \mathbf{k} \times \mathbf{u} = -\nabla p + RE\Theta \mathbf{r} + A(\nabla \times \mathbf{B}) \times \mathbf{B} + E\nabla^2 \mathbf{u},\tag{2.5}
$$

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$$
\nabla^2 - \frac{\partial}{\partial t} \mathbf{e} = \mathbf{u} \cdot \nabla \Theta - \mathbf{r} \cdot \mathbf{u},
$$
\n(2.6)

$$
\left|\nabla^2 - \frac{\partial}{\partial t}\right| \mathbf{B} = R_{\text{m}}(\mathbf{u} \cdot \nabla \mathbf{B} - \mathbf{B} \cdot \nabla \mathbf{u}). \tag{2.7}
$$

The Rayleigh number R, Ekman number E, Elsasser number Λ , and magnetic The Rayleigh number R , Ekman number E , Els
Reynolds number R_m are defined, respectively, as

Reynolds number
$$
R_m
$$
 are defined, respectively, as
\n
$$
R = \frac{\alpha \beta g_0 d^4}{\nu \kappa}, \qquad E = \frac{\nu}{2\Omega d^2}, \qquad A = \frac{B_0^2}{2\Omega \mu \rho \lambda}, \qquad R_m = \frac{\mathcal{U}d}{\lambda}, \qquad (2.8)
$$
\nwhere \mathcal{U} is a typical amplitude of the convection-driven flow.
\nConsider solutions for the magnetoconvection problem, in which a magnetic field is

EVALUATE:

Let us a typical amplitude of the convection-driven flow.

Consider solutions for the magnetoconvection problem, in which a magnetic field is

posed on the system. In magnetoconvection, the Elsasser number A is Consider solutions for the magnetoconvection problem, in which a magnetic field is imposed on the system. In magnetoconvection, the Elsasser number Λ is determined Consider solutions for the magnetoconvection problem, in which a magnetic field is
imposed on the system. In magnetoconvection, the Elsasser number Λ is determined
by the strength of the imposed magnetic field, whereas imposed on the system. In magnetoconvection, the Elsasser number Λ is determined
by the strength of the imposed magnetic field, whereas for the full dynamo problem,
the magnetic field is self-generated and its typical by the strength of the imposed magnetic field, whereas for the full dynamo problem,
the magnetic field is self-generated and its typical strength is determined by the
solution. The Elsasser number could, therefore, be dis e magnetic field is self-generated and its typical strength is determined by the
lution. The Elsasser number could, therefore, be dispensed with (by setting $B_0 = 2\Omega\mu\rho\lambda$) in the dynamo problem, but this is not helpful solution. The Elsasser number could, therefore, be dispensed with (by setting $B_0 = \sqrt{2\Omega\mu\rho\lambda}$) in the dynamo problem, but this is not helpful for magnetoconvection because we wish to study the response of the system t $\sqrt{2\Omega\mu\rho\lambda}$ in the dynamc
because we wish to study
imposed magnetic field.
Next, consider solutions because we wish to study the response of the system to different amplitudes of the imposed magnetic field.
Next, consider solutions to the kinematic dynamo problem, in which the fluid veloc-

imposed magnetic field.
Next, consider solutions to the kinematic dynamo problem, in which the fluid velocity is imposed. The magnetic Reynolds number R_m is determined by the strength of the imposed velocity, whereas fo Next, consider solutions to the kinematic dynamo problem, in which the fluid velocity is imposed. The magnetic Reynolds number R_m is determined by the strength of the imposed velocity, whereas for the full dynamo proble the imposed velocity, whereas for the full dynamo problem it is determined by the solution. In the kinematic dynamo problem, we wish to determine the flow speed the imposed velocity, whereas for the full dynamo problem it is determined by the solution. In the kinematic dynamo problem, we wish to determine the flow speed required to generate magnetic field, making R_m the importa solution. In the kinematic dynamo problem, we wish to determine the flow speed
required to generate magnetic field, making R_m the important external parameter,
but, like the Elsasser number, it can be dispensed with for required to generate magnetic field, making R_m the important external parameter,
but, like the Elsasser number, it can be dispensed with for the full dynamo problem.
With the scaling for the velocity used to form the di but, like the Elsasser number, it can be dispensed with for the full dynamo problem.
With the scaling for the velocity used to form the dimensionless equation of motion (2.5), the flow strength is $\mathcal{U} = \kappa/d$ and the ma With the scaling for th (2.5) , the flow strength
the Roberts number,

$$
R_{\rm m} = \kappa/\lambda = q,\tag{2.9}
$$

a property of the fluid.

a property of the fluid.
The kinematic dynamo and magnetoconvection are parallel simplifications of the
full dynamo problem: in the first, **u** is fixed and (2.3) is solved for **B**, whereas in the
second **B** is fixed and (The kinematic dynamo and magnetoconvection are parallel simplifications of the full dynamo problem: in the first, **u** is fixed and (2.3) is solved for **B**, whereas in the second, **B** is fixed and (2.1) – (2.2) are sol The kinematic dynamo and magnetoconvection are parallel simplifications of the full dynamo problem: in the first, **u** is fixed and (2.3) is solved for **B**, whereas in the second, **B** is fixed and (2.1) – (2.2) are solved for **u**. Both problems can be expected to reveal some of the character of t second, **B** is fixed and (2.1) – (2.2) are solved for **u**. Both problems can be expected to reveal some of the character of the full dynamo problem, but they differ from each other in important respects. The kinematic p E ROYAL other in important respects. The kinematic problem linearizes equation (2.3) (in **B**), yet it remains the correct equation for the full dynamo, whose solution must still satisfy (2.3) . Equations $(2.1)-(2.2)$ remain non yet it remains the correct equation for the full dynamo, whose solution must still yet it remains the correct equation for the full dynamo, whose solution must still satisfy (2.3) . Equations (2.1) – (2.2) remain nonlinear even when **B** is fixed; many of the results quoted are for marginal or weakly satisfy (2.3) . Equations (2.1) – (2.2) remain nonlinear even when **B** is fixed; many of the results quoted are for marginal or weakly nonlinear convection in which the flow is weak and the nonlinear terms small. No su the results quoted a
is weak and the non
dynamo problem.
In this paper we weak and the nonlinear terms small. No such restriction applies to the kinematic
namo problem.
In this paper, we simplify the equations further by taking the Roberts number to
unity and the Prandtl number to be infinite:

dynamo problem.
In this paper, we simplify the equations further by taking the Roberts number to
be unity and the Prandtl number to be infinite:

number to be infinite:
\n
$$
q = \frac{\kappa}{\lambda} = 1, \qquad Pr = \frac{\nu}{\kappa} = \infty.
$$
\n(2.10)

 $q = \frac{\kappa}{\lambda} = 1,$ $Pr = \frac{\nu}{\kappa} = \infty.$ (2.10)
Neither choice applies to the core directly, but we justify the first because turbulence
is expected to act to equalize the diffusivities and the second because small Prandtl Neither choice applies to the core directly, but we justify the first because turbulence
is expected to act to equalize the diffusivities and the second because small Prandtl *Phil. Trans. R. Soc. Lond.* A (2000)

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Scaledisparitiesinthegeodynamo ⁹⁰³

number flow is characterized by rapid time variations that are not observed in the geomagnetic record. We set $\eta = r_i/r_o = 0.4$ for our analysis throughout the paper.
The buoyancy-driven magnetohydrodynamic problem involves *IATHEMATICAL,
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k ENGINEERING
CIENCES* number flow is characterized by rapid time variations that are not observed in the geomagnetic record
The buoyancy-di
 $(2.5)-(2.7)$ and The buoyancy-driven magnetohydrodynamic problem involves solving equations $(2.5)-(2.7)$ and $\nabla \cdot \mathbf{u} = 0,$ (2.11 *a*)
(3.11 *a*)

$$
\nabla \cdot \mathbf{u} = 0,\tag{2.11 } a)
$$

$$
\nabla \cdot \mathbf{u} = 0,
$$
\n(2.11 *a*)\n
$$
\nabla \cdot \mathbf{B} = 0,
$$
\n(2.11 *b*)

together with appropriate boundary conditions for **u**, **B** and Θ . We assume that the inner and outer bounding spherical surfaces of the Earth's core are stress free and impenetrable,

$$
\frac{\partial (u_{\phi}/r)}{\partial r} = \frac{\partial (u_{\theta}/r)}{\partial r} = u_r = 0,
$$
\n(2.12)

 $\frac{\partial (u\psi/\nu)}{\partial r} = \frac{\partial (u\psi/\nu)}{\partial r} = u_r = 0,$ (2.12)
where (u_r, u_θ, u_ϕ) are the components of velocity in spherical polar coordinates.
Stress-free conditions give weaker boundary layers than rigid boundary conditions: where $(u_r, u_{\theta}, u_{\phi})$ are the components of velocity in spherical polar coordinates.
Stress-free conditions give weaker boundary layers than rigid boundary conditions:
note that the type of the velocity boundary condition where $(u_r, u_{\theta}, u_{\phi})$ are the components of velocity in spherical polar coordinates.
Stress-free conditions give weaker boundary layers than rigid boundary conditions:
note that the type of the velocity boundary condition Stress-free conditions give weaker boundary layers than rigid boundary conditions:
note that the type of the velocity boundary condition does not make a leading-order
contribution when E is sufficiently small (Roberts 1 note that the type of the velocity boundary condition does not make a leading-order contribution when E is sufficiently small (Roberts 1965; Zhang $\&$ Jones 1993; Fearn 1979). We also assume that both inner core and ma contribution when E is sufficiently small (Roberts 19
1979). We also assume that both inner core and m
insulating and thermally conducting, which yields conducting, which yields
 $\mathbf{r} \cdot (\nabla \times \mathbf{B}) = \Theta = 0,$ [B] = 0, (2.13)

$$
\mathbf{r} \cdot (\nabla \times \mathbf{B}) = \Theta = 0, \qquad [\mathbf{B}] = 0,
$$
 (2.13)

on the inner and outer bounding spherical surfaces, where [] denotes the jump across the bounding surfaces. This model does not include the potential stabilizing effect of
the bounding surfaces. This model does not include the potential stabilizing effect of
an electrically conducting inner core (Hollerba on the inner and outer bounding spherical surfaces, where [·] denotes the jump across
the bounding surfaces. This model does not include the potential stabilizing effect of
an electrically conducting inner core (Hollerbac the bounding surfaces. This model does not include the potential stabilizing effect of
an electrically conducting inner core (Hollerbach & Jones 1993, 1995). The numerical
methods employed are described in Gubbins & Zhang

an electrically conducting inner core (Hollerbach & Jones 1993, 1995). The numerical
methods employed are described in Gubbins & Zhang (1993), Gubbins *et al.* (2000*a*),
and papers cited therein.
In order to provide an e In order to provide an example of the scaling disparities, we have solved four difand papers cited therein.
In order to provide an example of the scaling disparities, we have solved four dif-
ferent related convection problems in this paper for various values of E, R and A: the
problems of linear and n In order to provide an example of the scaling disparities, we have solved four dif-
ferent related convection problems in this paper for various values of E, R and A: the
problems of linear and nonlinear convections (2.5) ferent related convection problems in this paper for various values of E, R and A: the problems of linear and nonlinear convection with $\Lambda = 0$ governed by equations (2.5), (2.6) and (2.11 a); and the problems of linear a problems of linear and nonlinear convection with $\Lambda = 0$ governed by equations (2.5), (2.6) and (2.11 *a*); and the problems of linear and nonlinear convection in the presence of an imposed magnetic field governed by equa (2.6) and (2.11 *a*); and the problems of linear and nonlinear convection in the presence of an imposed magnetic field governed by equations (2.5)–(2.7), (2.11 *a*) and (2.11 *b*). We also use solutions to the kinematic p

ence of an imposed magnetic field governed by equations $(2.5)-(2.7)$, $(2.11 a)$ and $(2.11 b)$. We also use solutions to the kinematic problem obtained by solving (2.7) with a parametrized flow containing some of the char $(2.11 b)$. We also use solutions to the kinematic problem obtained by solving (2.7) with a parametrized flow containing some of the characteristics of core convection to illustrate the variation in flow speed required t with a parametrized flow contaillustrate the variation in flow
with slightly different forms. with slightly different forms.
3. Spatial, temporal and amplitude scales with a weak field

When the dynamic effect of a magnetic field is sufficiently small, the Lorentz force may be neglected, decoupling the equations of motion and induction. There are
then two fundamentally different types of convection. The first takes the form of When the dynamic effect of a magnetic field is sufficiently small, the Lorentz force
may be neglected, decoupling the equations of motion and induction. There are
then two fundamentally different types of convection. The f may be neglected, decoupling the equations of motion and induction. There are
then two fundamentally different types of convection. The first takes the form of
thermal inertial waves, which oscillate so fast that viscosity The two fundamentally different types of convection. The first takes the form of thermal inertial waves, which oscillate so fast that viscosity may be neglected to leading order to give the Poincaré equation in a rotating thermal inertial waves, which oscillate so fast that viscosity may be neglected to leading order to give the Poincaré equation in a rotating spherical system (Zhang 1994, 1995b). Viscosity usually plays a purely dissipative role and the limit $E \to 0$ is regular. Only at the next order of approximation d 1994, 1995b). Viscosity usually plays a purely dissipative role and the limit $E \to 0$ is
regular. Only at the next order of approximation does the buoyancy force maintain
convection against weak viscous dissipation, which regular. Only at the next order of approximation does the buoyancy force maintain
convection against weak viscous dissipation, which takes place in Ekman boundary
layers. However, this type of convection, which is associat layers. However, this type of convection, which is associated with small Prandtl
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number, is unlikely to be important or relevant to dynamo action in the Earth's core
simply because of the short time-scale of the convective flow. number, is unlikely to be important or relevant to dynamo actions imply because of the short time-scale of the convective flow.
We shall focus on the second form of convection (Roberts 1968) simply because of the short time-scale of the convective flow.
We shall focus on the second form of convection (Roberts 1968; Busse 1970; see also

Jones *et al*. 2000), associated with large Prandtl number. It is slowly oscillatory and We shall focus on the second form of convection (Roberts 1968; Busse 1970; see also
Jones *et al.* 2000), associated with large Prandtl number. It is slowly oscillatory and
the inertial terms do not enter into the leading Jones *et al.* 2000), associated with large Prandtl number. It is slowly oscillatory and the inertial terms do not enter into the leading-order problem. The most significant feature of the convection is the role of viscos feature of the convection is the role of viscosity: it provides the necessary frictional forces to offset that part of the Coriolis force $2Qk \times u$ that cannot be balanced by feature of the convection is the role of viscosity: it provides the necessary frictional
forces to offset that part of the Coriolis force $2Qk \times u$ that cannot be balanced by
the pressure gradient $-\nabla p/\rho$. Convection can forces to offset that part of the Coriolis force 2Ω **k** \times **u** that cannot be balanced by the pressure gradient $-\nabla p/\rho$. Convection cannot take place without a large frictional force; in this sense, the role of visc the pressure gradient $-\nabla p/\rho$. Convection cannot take place without a large frictional force; in this sense, the role of viscosity is inverted from the usual one of inhibiting or preventing convection (by providing a si force; in this sense, the role of viscosity is inverted from the usual one of inhibiting or
preventing convection (by providing a sink for potential energy that would otherwise
be converted to kinetic energy), to an essent preventing convection (by providing a sink for potential energy that would
be converted to kinetic energy), to an essential force that allows convection
by breaking the Proudman–Taylor constraint imposed by the rotation.
 converted to kinetic energy), to an essential force that allows convection to occur
breaking the Proudman-Taylor constraint imposed by the rotation.
Application of the operator $\mathbf{r} \cdot \nabla \times$ to (2.5) yields the radial

by breaking the Proudman–Taylor constraint imposed by the rotation.
Application of the operator $\mathbf{r} \cdot \nabla \times$ to (2.5) yields the radial component of the vorticity equation

$$
-\mathbf{r} \cdot \frac{\partial \mathbf{u}}{\partial z} = E \mathbf{r} \cdot \nabla \times \nabla \times \nabla \times \mathbf{u},\tag{3.1}
$$

where (s, ϕ, z) are cylindrical polar coordinates with z along k. The Proudmanwhere (s, ϕ, z) are cylindrical polar coordinates with z along **k**. The Proudman-
Taylor theorem requires changes of **u** and Θ to be small at low viscosity, so that $\partial/\partial z$ is $O(1)$. The primary balance in the equatio where (s, ϕ, z) are cylindrical polar coordinates with z along **k**. The Proudman-
Taylor theorem requires changes of **u** and Θ to be small at low viscosity, so that $\partial/\partial z$ is $O(1)$. The primary balance in the equatio $\partial/\partial z$ is $O(1)$. The primary balance in the equation of motion is between pressure and Coriolis forces, but in the vorticity equation there is no pressure and the balance must be struck with the viscous forces. This is achieved at small Ekman number and Coriolis forces, but in the vorticity equation there is no pressure and the balance
must be struck with the viscous forces. This is achieved at small Ekman number
by small length-scales in the s and ϕ directions.), must be struck with the viscous forces. This is achieved at small Ekman number
by small length-scales in the s and ϕ directions. $(\nabla \times)^3$ may be taken to be $O(m^3)$,
where m is the azimuthal wavenumber of convection. where *m* is the azimuthal wavenumber of convection. Equation (3.1) shows that the limit $E \to 0$ is singular because the wavelength of the convective flow goes to zero:

$$
m = O\left[\left|\frac{\nu}{2\Omega d^2}\right|^{-1/3}\right] = O(E^{-1/3}), \text{ as } E \to 0.
$$
 (3.2)

 $m = O$ $\sqrt{2\Omega d^2}$ $\sqrt{2Qd^2}$ $\sqrt{D} = O(E^{-1/9})$, as $E \to 0$. (3.2)
Equation (3.2) is one of the fundamental asymptotic laws that provide a basic frame-
work for understanding convection in a sphere **IATHEMATICAL,
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work for understanding convection in a sphere.
Note that equations (3.1) and (3.2) are valid at work for understanding convection in a sphere.
Note that equations (3.1) and (3.2) are valid at infinite Prandtl number regardless

work for understanding convection in a sphere.
Note that equations (3.1) and (3.2) are valid at infinite Prandtl number regardless
of the size of the Rayleigh number R . Thus, provided no small scales develop in
the z-di Note that equations (3.1) and (3.2) are valid at infinite Prandtl number regardless
of the size of the Rayleigh number R. Thus, provided no small scales develop in
the z-direction, the presence of large wavenumbers (s of the size of the Rayleigh number R . Thus, provided no small scales develop in the *z*-direction, the presence of large wavenumbers (small scales) will persist even in a strongly nonlinear regime. The fundamental laws the *z*-direction, the presence of large wavenumbers (small scales)
in a strongly nonlinear regime. The fundamental laws were first d
(1968) from his asymptotic theory for a rapidly rotating sphere,

from his asymptotic theory for a rapidly rotating sphere,
\n
$$
R_c = O(E^{-4/3})
$$
, $m_c = O(E^{-1/3})$, $\omega_c = O(E^{-2/3})$, as $E \to 0$, (3.3)

 $R_c = O(E^{-4/3})$, $m_c = O(E^{-1/3})$, $\omega_c = O(E^{-2/3})$, as $E \to 0$, (3.3)
where R_c is the critical Rayleigh number, the smallest value of the Rayleigh number
at which convection can take place and m_c and ω_c are the correspondin where R_c is the critical Rayleigh number, the smallest value of the Rayleigh number
at which convection can take place, and m_c and ω_c are the corresponding wavenumber
and frequency of convection (see also Soward 19 where R_c is the critical Rayleigh number, the smallest value of the Rayleigh number at which convection can take place, and m_c and ω_c are the corresponding wavenumber and frequency of convection (see also Soward 19 **O** and frequency of convection (see also Soward 1977; Zhang 1991, 1992; Jones *et al.*
2000).
In (3.3), the coefficients of the asymptotic laws are functions of the Prandtl num-

2000).
In (3.3), the coefficients of the asymptotic laws are functions of the Prandtl num-
ber Pr . It was shown by Busse (1970), based on a local asymptotic analysis, that
convection with symmetry In (3.3) , the coefficients of t
ber Pr . It was shown by Buss
convection with symmetry

$$
(u_r, u_\theta, u_\phi)(r, \theta, \phi) = (u_r, -u_\theta, u_\phi)(r, \pi - \theta, \phi), \qquad \Theta(r, \theta, \phi) = \Theta(r, \pi - \theta, \phi),
$$
\n(3.4)

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Scaledisparitiesinthegeodynamo ⁹⁰⁵

Table 1. An example of the scale disparities for $E = 10^{-5}$

Table 1. An example of the scale disparities for $E = 10^{-5}$
(The critical Rayleigh number R_c , the corresponding azimuthal wavenumber m_c and the frequency ω_c of convection in a rapidly rotating spherical shell with (The critical Rayleigh number R_c , the corresponding azimuthal wavenumber m_c and the frequency ω_c of convection in a rapidly rotating spherical shell with or without the effect of a magnetic field for $r_c/r_c = 0.4$ Th (The critical Rayleigh number R_c , the corresponding azimuthal wavenumber m_c and the frequency ω_c of convection in a rapidly rotating spherical shell with or without the effect of a magnetic field for $r_i/r_o = 0.4$. T quency ω_c of convection in a rapidly rotating spherical shell with or without the effect of a magnetic field for $r_i/r_o = 0.4$. The parameter ϵ_P is related to the form of the basic magnetic field in the magnetoconvect field in the magnetoconvection problem defined by (4.1). The time-scale of the frequency is based on the magnetic diffusion time-scale with $q = 1$.)

Figure 1. The small spatial scale and short time-scale convection in a rapidly rotating spherical fluid shell with $r_i/r_o = 0.4$ at $E = 10^{-5}$ with no magnetic field $(A = 0)$, see also table 1). Shown are streamlines of the fluid shell with $r_i/r_0 = 0.4$ at $E = 10^{-5}$ with no magnetic field $(A = 0$, see also table 1). Shown are streamlines of the convective flow on the outer spherical surface viewed from a 30[°] angle (a) and viewed from the N are streamlines of the convective flow on the outer spherical surface viewed from a 30° angle (a).

occurs at lowest R_c and is, therefore, physically realizable. The multiplicative conoccurs at lowest R_c and is, therefore, physically realizable. The multiplicative constants in these asymptotic laws can be found by extrapolating the results of numerical calculations with finite E . For a rotating sph occurs at lowest R_c and is, therefore, physically realizable. The multiplicative constants in these asymptotic laws can be found by extrapolating the results of numerical calculations with finite E. For a rotating spher

ulations with finite E. For a rotating spherical shell with
$$
r_i/r_o = 0.4
$$
 they are
\n $R_c = 1.63E^{-4/3}$, $m_c = 0.74E^{-1/3}$, $\omega_c = -0.25E^{-2/3}$, as $E \to 0$. (3.5)

 $R_c = 1.63E^{-4/3}$, $m_c = 0.74E^{-1/3}$, $\omega_c = -0.25E^{-2/3}$, as $E \rightarrow 0$. (3.5)
An example of the convection solution at $E = 10^{-5}$, which shows streamlines on
equiter spherical surface is displayed in figure 1. More details are An example of the convection solution at $E = 10^{-5}$, which shows streamlines on the outer spherical surface, is displayed in figure 1. More details are given in table 1. These solutions are new although their behaviour wa An example of the convection solution at $E = 10^{-5}$, which shows streamlines on the outer spherical surface, is displayed in figure 1. More details are given in table 1. These solutions are new, although their behaviour w the outer spherical surface, is displayed in figure 1. More details are given in table 1.
These solutions are new, although their behaviour was already qualitatively well
understood: they were computed for this paper in o These solutions are new, although their behaviour was already qualitatively well
understood: they were computed for this paper in order to establish the asymptotic
behaviour at small E . The convection is in the form of understood: they were computed for this paper in order to establish the asymptotic
behaviour at small E . The convection is in the form of nearly two-dimensional rolls
(Busse's columnar rolls) aligned with the axis of ro behaviour at small E. The convection is in the (Busse's columnar rolls) aligned with the axis
at higher latitudes with a weak phase shift.
Nonlinear calculations have been performe (Busse's columnar rolls) aligned with the axis of rotation and located and localized
at higher latitudes with a weak phase shift.
Nonlinear calculations have been performed to obtain the corresponding weakly

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nonlinear asymptotic law for the amplitude of convection, which is

for the amplitude of convection, which is
\n
$$
\mathcal{U} = 20.1(RE^{4/3} - 1.63)^{1/2}\lambda/d,
$$
\n(3.6)

 $U = 20.1(RE^{4/3} - 1.63)^{1/2}\lambda/d,$ (3.6)
where U is now defined precisely as the average speed of convection over the spherical
fluid shell V: where $\mathcal U$ is now $\mathfrak d$
fluid shell V :

$$
\mathcal{U}^2 = \int_V |\mathbf{u}|^2 \, dV \quad \bigg| \int_V dV. \tag{3.7}
$$

 $\mathcal{U} = \int_V |\mathbf{u}| dV \int_V dV.$ (3.7)
We know the approximate flow speed in the Earth's core, so it is useful to rearrange
(3.6) to give the corresponding Rayleigh number: We know the approximate flow speed in the Earth's (3.6) to give the corresponding Rayleigh number:

$$
R_U = \left[\left| \frac{\mathcal{U}d}{20.1\lambda} \right|^2 + 1.63 \right] E^{-4/3}.
$$
 (3.8)

 $n_U = \frac{1}{20.1\lambda} + 1.03E$ (3.8)
The corresponding dominant wavenumber and frequency are only slightly modified
the nonlinearity. Similarly, a weakly nonlinear asymptotic relation for the total The corresponding dominant wavenumber and frequency are only slightly modified
by the nonlinearity. Similarly, a weakly nonlinear asymptotic relation for the total
convective heat flux H at the outer spherical surface c by the nonlinearity. Similarly, a weakly nonlinear asymptotic relation for the total convective heat flux H at the outer spherical surface can also be obtained

$$
H = (4\pi r_0^2 K_{\rm T}) \frac{\Delta T}{d} 3.72 \times 10^{-2} (RE^{4/3} - 1.63), \tag{3.9}
$$

 $H = (4\pi r_0^2 K_T) \frac{1}{d} 3.72 \times 10^{-2} (RE^{4/3} - 1.63),$ (3.9)
where K_T is the thermal conductivity, and ΔT is the superadiabatic temperature
difference across the fluid shell. where $K_{\rm T}$ is the thermal conductifierence across the fluid shell.
These asymptotic laws can be These asymptotic laws can be used to extrapolate the results to low Ekman num-
These asymptotic laws can be used to extrapolate the results to low Ekman num-
r when the length-scales are much too small to be simulated num

difference across the fluid shell.
These asymptotic laws can be used to extrapolate the results to low Ekman num-
ber, when the length-scales are much too small to be simulated numerically. Molecu-These asymptotic laws can be use
ber, when the length-scales are much
lar diffusivities $(\nu = \kappa = 10^{-6} \text{ m}^2 \text{ s}^{-1})$
values $(\nu = \kappa = \lambda)$ give $E \approx 10^{-10}$ (s^{-1}) g l to extrapolate the results to low Ekman num-
too small to be simulated numerically. Molecu-
) give $E \approx 10^{-15}$ in the Earth's core, turbulent
faking $d = 2 \times 10^6$ m and $\lambda = 1$ m² s⁻¹ ($a = 1$) ber, when the length-scales are much too small to be simulated numerically. Molecular diffusivities $(\nu = \kappa = 10^{-6} \text{ m}^2 \text{ s}^{-1})$ give $E \approx 10^{-15}$ in the Earth's core, turbulent values $(\nu = \kappa = \lambda)$ give $E \approx 10^{-10}$. Ta s^{-1} $(q$ lar diffusivities $(\nu = \kappa = 10$
values $(\nu = \kappa = \lambda)$ give E and E = 10⁻¹⁵

or
$$
E = 10^{-15}
$$

\n $R_c = 1.6 \times 10^{20}$, $L = 27$ m, $T = 10^4$ s, for $E = 10^{-15}$, (3.10)

 $R_c = 1.6 \times 10^{20}$, $L = 27 \text{ m}$, $T = 10^4 \text{ s}$, for $E = 10^{-15}$, (3.10)
where L is the horizontal scale of convection rolls and T is the period of oscillation
(or azimuthal drift) both of which are extremely small and f where *L* is the horizontal scale of convection rolls and *T* is the period of oscill (or azimuthal drift), both of which are extremely small, and, for $E = 10^{-10}$,

(or azimuthal drift), both of which are extremely small, and, for
$$
E = 10^{-10}
$$
,
\n $R_c = 3.5 \times 10^{13}$, $L = 1.3$ km, $T = 3.4 \times 10^6$ s, for $E = 10^{-10}$, (3.11)
\nwhich still gives small-scale, rapidly fluctuating convection for core parameters.
\nIf we further assume that the weakly nonlinear expressions (3.6) (3.9) remain

ich still gives small-scale, rapidly fluctuating convection for core parameters.
If we further assume that the weakly nonlinear expressions (3.6) , (3.9) remain
lid for strongly nonlinear convection, we may estimate th which still gives small-scale, rapidly fluctuating convection for core parameters.
If we further assume that the weakly nonlinear expressions (3.6) , (3.9) remain valid for strongly nonlinear convection, we may estimat If we further assume that the weakly nonlinear expressions (3.6), (3.9) remain valid for strongly nonlinear convection, we may estimate the size of the Rayleigh number and the convective heat flux. Taking a typical core f valid for strongly nonlinear convection, we may estimate the size of the Rayleigh
number and the convective heat flux. Taking a typical core flow speed estimated
from geomagnetic secular variation to be $U = 10^{-4}$ m s⁻¹ number and the convective heat flux. Taking a typical core flow speed estimated
from geomagnetic secular variation to be $\mathcal{U} = 10^{-4}$ m s⁻¹, we can use equation (3.8)
to estimate the Rayleigh number $R_U = 10^{22}$ for from geomagnetic secular variation to be $\mathcal{U} = 10^{-4}$ m s⁻¹, we can use equation (3.8)
to estimate the Rayleigh number $R_U = 10^{22}$ for $E = 10^{-15}$ and $R_U = 2.2 \times 10^{15}$
for $E = 10^{-10}$. Note that in both cases the for $E = 10^{-10}$. Note that in both cases the required Rayleigh number is 60 times $T_E = 10^{-10}$. Note that in both cases the required Rayleigh number is 60 times itical, because both R and R_c scale with the same power of the Ekman number.
These numerical estimates are presented here to demonstrate the

critical, because both R and R_c scale with the same power of the Ekman number.
These numerical estimates are presented here to demonstrate the extremes of scale
that arise in non-magnetic convection. Their relevance f These numerical estimates are presented here to demonstrate the extremes of scale
that arise in non-magnetic convection. Their relevance for the Earth's core is dis-
cussed in $\S 7$; they are only given here for compariso that arise in non-magnetic convection. Their relevance for the Earth's core is discussed in $\S 7$; they are only given here for comparison with magnetoconvection and full dynamo calculations.

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4. Spatial, temporal and amplitude scales with a strong toroidal field

4. Spatial, temporal and amplitude scales with a strong toroidal field
We now assume the fluid is permeated by an axisymmetric magnetic field with both
toroidal and poloidal parts: The space of the poloidal and poloidal parts:

$$
\mathbf{B} = B_0(\epsilon_P \mathbf{B}_P + \mathbf{B}_T),\tag{4.1}
$$

 $\mathbf{B} = B_0(\epsilon_P \mathbf{B}_P + \mathbf{B}_T),$ (4.1)
where \mathbf{B}_T is the toroidal part and \mathbf{B}_P the poloidal part, scaled so that $|\mathbf{B}_P|_{\text{max}} = |\mathbf{B}_T|_{\text{max}} = 1$. Further, we assume that **B** has dipole symmetry (see, for exam where B_T is the toroidal part and B_P the poloidal part, scaled so that $|B_P|_{\text{max}} = |B_T|_{\text{max}} = 1$. Further, we assume that **B** has dipole symmetry (see, for example, Gubbins & Zhang 1993) where B_T is the toroidal p
 $|B_T|_{max} = 1$. Further, we
Gubbins & Zhang 1993)

$$
(B_r, B_\theta, B_\phi)(r, \theta, \phi) = (-B_r, B_\theta, -B_\phi)(r, \pi - \theta, \phi).
$$
\n(4.2)

 $(B_r, B_\theta, B_\phi)(r, \theta, \phi) = (-B_r, B_\theta, -B_\phi)(r, \pi - \theta, \phi).$ (4.2)
This imposed field is supposed to represent the main dynamo-generated field in the
Earth's core, but the equations are only self-consistent if we suppose the field is This imposed field is supposed to represent the main dynamo-generated field in the Earth's core, but the equations are only self-consistent if we suppose the field is maintained by some external source, because we are not This imposed field is supposed to represent the main dynamo-generated field in the Earth's core, but the equations are only self-consistent if we suppose the field is maintained by some external source, because we are not Earth's core, but the equations are only self-consistent if we suppose the fiel maintained by some external source, because we are not solving the full dynapoleautions. Any axisymmetric toroidal field B_T can be written equations. Any axisymmetric toroidal field B_T can be written in the form

$$
\mathbf{B}_{\mathrm{T}} = -\sum_{l,n} g_{ln} \frac{\partial G_{ln}(\theta, r)}{\partial \theta} \hat{\phi},\tag{4.3}
$$

where g_{ln} are real constants and $G_{ln}(\theta, r)$ are solutions of Helmholtz's equation

and
$$
G_{ln}(\theta, r)
$$
 are solutions of Helmholtz's equation

$$
(\beta_{ln}^2 + \nabla^2)G_{ln}(\theta, r) = 0,
$$
(4.4)

which have the form

he form
\n
$$
G_{ln}(\theta, r) = P_l(\cos \theta) [j_l(r_i \beta_{ln}) n_l(r \beta_{ln}) - j_l(r \beta_{ln}) n_l(r_i \beta_{ln})],
$$
\n(4.5)

 $G_{ln}(\theta, r) = P_l(\cos \theta) [j_l(r_i \beta_{ln}) n_l(r \beta_{ln}) - j_l(r \beta_{ln}) n_l(r_i \beta_{ln})],$ (4.5)
where $P_l(\cos \theta)$ is the Legendre function, $j_l(r \beta_{ln})$ and $n_l(r \beta_{ln})$ are spherical Bessel
functions of the first and second kinds and the β_{ln} are determine where $P_l(\cos\theta)$ is the Legendre function, $j_l(r\beta_{ln})$ and $n_l(r\beta_{ln})$ are spherifunctions of the first and second kinds, and the β_{ln} are determined by functions of the first and second kinds, and the β_{ln} are determined by

$$
j_l(r_i\beta_{ln})n_l(r_o\beta_{ln}) - j_l(r_o\beta_{ln})n_l(r_i\beta_{ln}) = 0,
$$
\n(4.6)

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 $0 < \beta_{l1} < \beta_{l2} < \beta_{l3} < \cdots, \quad l = 1, 2, 3, \ldots$ (4.7)

Similarly, any poloidal magnetic field B_P may be written in the form

$$
\mathbf{B}_{\rm P} = \sum_{l,n} h_{ln} \left[\left| -r \nabla^2 H_{ln} + \frac{1}{r} \frac{\partial}{\partial r} r^2 \frac{\partial H_{ln}}{\partial r} \right| \hat{\mathbf{r}} + \frac{1}{r} \frac{\partial}{\partial \theta} \frac{\partial (r H_{ln})}{\partial r} \hat{\theta} \right], \tag{4.8}
$$

where h_{ln} are real constants and $H_{ln}(\theta, r)$ satisfies

and
$$
H_{ln}(\theta, r)
$$
 satisfies
\n
$$
(\xi_{ln}^2 + \nabla^2) H_{ln}(\theta, r) = 0,
$$
\n(4.9)

with ξ_{ln} being determined by

ermined by
\n
$$
j_l(r_i\xi_{ln})n_{l-1}(r_o\xi_{ln}) - j_{l-1}(r_o\xi_{ln})n_l(r_i\xi_{ln}) = 0,
$$
\n(4.10)

 $j_l(r_i\xi_{ln})n_{l-1}(r_o\xi_{ln}) - j_{l-1}(r_o\xi_{ln})n_l(r_i\xi_{ln}) = 0,$ (4.10)
with $0 < \xi_{l1} < \xi_{l2} < \xi_{l3} < \cdots$. We mimic the geomagnetic field, which is dominated
by the largest scales by choosing $l = 1$, $n = 1$ for the poloidal field and with $0 < \xi_{l1} < \xi_{l2} < \xi_{l3} < \cdots$. We mimic the geomagnetic field, which is dominated
by the largest scales, by choosing $l = 1$, $n = 1$ for the poloidal field and $l = 2$, $n = 1$
for the toroidal field. The problem of magn with $0 < \xi_{l1} < \xi_{l2} < \xi_{l3} < \cdots$. We mimic the geomagnetic field, which is dominated by the largest scales, by choosing $l = 1$, $n = 1$ for the poloidal field and $l = 2$, $n = 1$ for the toroidal field. The problem of magn for the toroidal field. The problem of magnetoconvection, although the magnetic *Phil. Trans. R. Soc. Lond.* A (2000)

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Figure 2. The large spatial scale and long time-scale convection in a rapidly rotating spherical fluid shell with $r_i/r_o = 0.4$ at $E = 10^{-5}$ with imposed toroidal magnetic field at $\Lambda = 8$, $\epsilon_P = 0$ (see also table 1). Vie Figure 2. The large spatial scale and long time-scale convection in a rapidly rotating spherical Figure 2. The large spatial scale and long time-scale
fluid shell with $r_i/r_o = 0.4$ at $E = 10^{-5}$ with impose
(see also table 1). Views (*a*) and (*b*) as in figure 1. $\overline{\sigma}$

(see also table 1). Views (*a*) and (*b*) as in figure 1.
field is imposed, contains many essential dynamic elements similar to those in mag-
netohydrodynamic dynamos (Proctor 1994: see also Fearn & Proctor 1983: Zhang & field is imposed, contains many essential dynamic elements similar to those in magnetohydrodynamic dynamos (Proctor 1994; see also Fearn & Proctor 1983; Zhang & Jones 1994: Zhang 1995a: Olson & Glatzmaier 1995, 1996). Jones 1994; Zhang 1995a; Olson & Glatzmaier 1995, 1996).
When a strong magnetic field is imposed onto the convection system all the time,

netohydrodynamic dynamos (Proctor 1994; see also Fearn & Proctor 1983; Zhang & Jones 1994; Zhang 1995*a*; Olson & Glatzmaier 1995, 1996).
When a strong magnetic field is imposed onto the convection system *all the time*, t the dynamical role of viscosity can be taken up by the magnetic force. This can easily

$$
\mathbf{r} \cdot \frac{\partial \mathbf{u}}{\partial z} \sim A \mathbf{r} \cdot ((\nabla \times \mathbf{B}) \times \mathbf{B}). \tag{4.11}
$$

Taking $\partial/\partial z = O(1)$ together with Ohm's law yields an estimate $L = O(dA)$, where A Taking $\partial/\partial z = O(1)$ together with Ohm's law yields an estimate $L = O(d\overline{A})$, where \overline{A} is the Elsasser number with B_0 based on an average magnetic field. For a sufficiently strong imposed field $\overline{A} = O(1)$ the f Taking $\partial/\partial z = O(1)$ together with Ohm's law yields an estimate $L = O(d\Lambda)$, where Λ is the Elsasser number with B_0 based on an average magnetic field. For a sufficiently strong imposed field, $\bar{\Lambda} = O(1)$, the fundame is the Elsasser number w
strong imposed field, \bar{A}
(3.3) are replaced by), $m_c = O(\bar{A}^{-1}),$), $\omega_c = O(1)$, as $E \to 0$. (4.12)

$$
R_c = O(E^{-1}), \quad m_c = O(\bar{A}^{-1}), \quad \omega_c = O(1), \quad \text{as } E \to 0.
$$
 (4.12)

 $R_c = O(E^{-1}), \quad m_c = O(\bar{\Lambda}^{-1}), \quad \omega_c = O(1), \quad \text{as } E \to 0.$ (4.12)
The convection is large scale and slowly varying on the diffusion time-scale, in
sharp contrast to non-magnetic convection (3.3) Furthermore, the Bayleigh number The convection is large scale and slowly varying on the diffusion time-scale, in sharp contrast to non-magnetic convection (3.3). Furthermore, the Rayleigh number required to initiate convection is much smaller $(O(E^{-1}))$ in The convection is large scale and slowly varying on the diffusion time-scale, in sharp contrast to non-magnetic convection (3.3). Furthermore, the Rayleigh number required to initiate convection is much smaller $(O(E^{-1}))$, i sharp contrast to non-magnetic convection (3.3). Furthermore, the Rayleigh number
required to initiate convection is much smaller $(O(E^{-1}))$, in contrast with $O(E^{-4/3})$
in (3.3), a factor of $E^{-1/3}$ or 10⁵ for molecular v both much smaller ($O(E^{-1})$, in contrast with O
for molecular values of the diffusivities).
he verified and values placed on the coeffic $E1$ quired to initiate convection is much smaller $(O(E^{-1}))$, in contrast with $O(E^{-4/3})$
(3.3), a factor of $E^{-1/3}$ or 10^5 for molecular values of the diffusivities).
Again, the asymptotic laws can be verified and values pla $)$

in (3.3), a factor of $E^{-1/3}$ or 10⁵ for molecular values of the diffusivities).
Again, the asymptotic laws can be verified and values placed on the coefficients by
numerical simulation at relatively large Ekman number Again, the asymptotic laws can be verified and values placed on the coefficients by
numerical simulation at relatively large Ekman numbers. For a purely toroidal field
with $\epsilon_{\rm P} = 0$ and $\Lambda = 10$ (which is based on the numerical simulation at relatively large Ekman numbers. For a purely toroidal field with $\epsilon_P = 0$ and $\Lambda = 10$ (which is based on the maximum value of $|B|$, equivalent to $\Lambda = O(1)$, we obtain ; $m_c = 1$, $\omega_c = -8.3$, as $E \to 0$, (4.13)

$$
R_c = 12E^{-1}
$$
, $m_c = 1$, $\omega_c = -8.3$, as $E \to 0$, (4.13)

which should be compared with (3.5) . An example of our convection solution for $E = 10^{-5}$ and $\Lambda = 8$ is shown in figure 2. It displays streamlines of the convective

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Scale disparities in the geodynamo
motions on the outer spherical surface, which should be compared with figure 1 with
 $\Lambda = 0$. Details of the relevant parameters for figure 2 are also given in table 1. otions on the outer spherical surface, which should be compared with figure 1 v = 0. Details of the relevant parameters for figure 2 are also given in table 1.
Nonlinear calculations at $\epsilon_{\text{D}} = 0$ (Zhang 1999) give the otions on the outer spherical surface, which should be compared with figure 1 with = 0. Details of the relevant parameters for figure 2 are also given in table 1.
Nonlinear calculations at $\epsilon_{\rm P} = 0$ (Zhang 1999) give t

 $\Lambda = 0$. Details of the relevant parameters for figure 2 are also given in table 1.
Nonlinear calculations at $\epsilon_{\rm P} = 0$ (Zhang 1999) give the corresponding weakly
nonlinear asymptotic relation for the finite amplitude nonlinear asymptotic relation for the finite amplitude of magnetoconvection corresponding to (3.6) : $\mathcal{U} = 3(RE - 12)^{1/2} \lambda/d, \text{ as } E \to 0.$ (4.14)

$$
\mathcal{U} = 3(RE - 12)^{1/2}\lambda/d, \text{ as } E \to 0.
$$
 (4.14)

 $\mathcal{U} = 3(RE - 12)^{1/2}\lambda/d$, as $E \to 0$.
Similarly, the relation corresponding to (3.9) for the heat flux *H* is:

on corresponding to (3.9) for the heat flux H is:
\n
$$
H = 1.01 \times 10^{-2} (4\pi r_o^2 K_T) \Delta T (RE - 12) / d.
$$
\n(4.15)

With both linear and nonlinear asymptotic relations for $E \to 0$, we can again esti-With both linear and nonlinear asymptotic relations for $E \to 0$, we can again estimate quantities in the Earth's core by extrapolation using equations (4.13) and (4.14) when the magnetic fields are strong ($\bar{A} = O(1)$). With both linear and nonlinear asymptotic relations for $E \to 0$, we can again estimate quantities in the Earth's core by extrapolation using equations (4.13) and (4.14) when the magnetic fields are strong ($\bar{A} = O(1)$). \bullet ties, s,
 $R_c = 1.2 \times 10^{16}$, $L = d = 2 \times 10^6$ m, $T = 4 \times 10^{18}$ s, for $E = 10^{-15}$,

$$
R_c = 1.2 \times 10^{16}
$$
, $L = d = 2 \times 10^6$ m, $T = 4 \times 10^{18}$ s, for $E = 10^{-15}$, (4.16)
which should be compared with (3.10); and, for turbulent values,

which should be compared with (3.10); and, for turbulent values,

$$
R_c = 1.2 \times 10^{11}
$$
, $L = d = 2 \times 10^6$ m, $T = 4 \times 10^{12}$ s, for $E = 10^{-10}$, (4.17)

which should be compared with (3.11).

The Rayleigh number required to produce a typical core flow speed of $\mathcal{U} =$ which should
The Raylei
 10^{-4} m s⁻¹ is is

$$
R_U = \left[\left| \frac{\mathcal{U}d}{3\lambda} \right|^2 + 12, E^{-1}, \right] \tag{4.18}
$$

 $R_U = \prod_{i=1}^{N_U} \frac{1}{3\lambda} + 12 \prod_{i=1}^{N} E^{-1}$, (4.18)
which gives $R_U = 1.3 \times 10^{18}$ for $E = 10^{-15}$ and 1.3×10^{13} for $E = 10^{-10}$. Both
Rayleigh numbers are 100 times critical which gives $R_U = 1.3 \times 10^{18}$ for $E = 10$
Rayleigh numbers are 100 times critical.
The interpretation of these numbers for ich gives $R_U = 1.3 \times 10^{18}$ for $E = 10^{-15}$ and 1.3×10^{13} for $E = 10^{-10}$. Both ayleigh numbers are 100 times critical.
The interpretation of these numbers for the Earth's core is postponed to $\S 7$. We uphasize her

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The interpretation of these numbers for the Earth's core is postponed to §7. We
emphasize here the huge contrast between the spatial, temporal and amplitude scales
obtained at $\bar{$ The interpretation of these numbers for the Earth's core is postponed to $\S 7$. We emphasize here the huge contrast between the spatial, temporal and amplitude scales obtained at $\overline{\Lambda} = 0$ (equations (3.10) and (3.11)) emphasize here the huge contrast between the spatial, temporal and amplitude scales
obtained at $\bar{A} = 0$ (equations (3.10) and (3.11)) and at $\bar{A} = O(1)$ (equations (4.16)
and (4.17)). The smaller the Ekman number, the obtained at $\Lambda = 0$ (equations (3.10) and (3.11)) and at $\Lambda = O(1)$ (equations (4.16) and (4.17)). The smaller the Ekman number, the larger the scale disparities; they are a fundamental characteristic of magnetohydrodynami

and (4.17)). The smaller the Ekman number, the larger the scale disparities; they
are a fundamental characteristic of magnetohydrodynamics in the Earth's fluid core.
It is worth noting that these scale disparities are rem are a fundamental characteristic of magnetohydrodynamics in the Earth's fluid core.
It is worth noting that these scale disparities are removed almost entirely by hyperviscosity at $E \geq 10^{-6}$ (Zhang & Jones 1997): the problem is simply not addressed by the current generation of numerical dynamo simulations employing hyperviscosity.

Experimention of numerical dynamo simulations employing hyperv
5. Spatial, temporal and amplitude scales with
the effect of a poloidal field , temporal and amplitude scale
the effect of a poloidal field

A strong-field dynamo satisfying the Taylor constraint (1.2) is *stable* if small pertur-
hations lead to small changes in the system. Perhaps, intrinsic instability explains Δ strong-field dynamo satisfying the Taylor constraint (1.2) is *stable* if small pertur-
bations lead to small changes in the system. Perhaps, intrinsic instability explains
why one cannot obtain a strong-field dynamo A strong-field dynamo satisfying the Taylor constraint (1.2) is *stable* if small perturbations lead to small changes in the system. Perhaps, intrinsic instability explains why one cannot obtain a strong-field dynamo nu **O** bations lead to small changes in the system. Perhaps, intrinsic instability explains why one cannot obtain a strong-field dynamo numerically (see, for example, Fearn $\&$ Proctor 1987). We now illustrate a possible in why one cannot obtain a strong-field dynamo numerically (see, for example, Fearn $\&$ $\frac{1}{\circ}$ component ($\epsilon_P \neq 0$).
First, we investigated the linear instability of the magnetoconvective system by (linear and nonlinear) with an imposed magnetic field (4.1) that includes a poloidal

including a small poloidal magnetic field while keeping the toroidal field unchanged

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Figure 3. The scaled critical Rayleigh number, ER_c , and the corresponding drift rate C are plotted against ϵ_P at $E = 10^{-4}$. The primary features of this graph are independent of the Ekman number provided it is small plotted against $\epsilon_{\rm P}$ at $E = 10^{-4}$. The primary features of this graph are independent of the Ekman number, provided it is small, because of the asymptotic forms (4.13). Reproduced from Zhang & Gubbins (2000) with the permission of Blackwell Science Ltd.

(Zhang & Gubbins 2000). We calculated about 30 solutions at small Ekman number while increasing ϵ_P gradually from zero. We found a dramatic fall in critical Rayleigh number with ϵ_P (figure 3): the product ER_c falls by a factor of 10 as ϵ_P increases from zero to 0.07, still a very small poloidal field. When the Ekman number is very number with ϵ_P (figure 3): the product ER_c falls by a factor of 10 as ϵ_P increases
from zero to 0.07, still a very small poloidal field. When the Ekman number is very
small, this represents a huge fall in R_c itse from zero to 0.07, still a very small poloidal field. When the Ekman number is very small, this represents a huge fall in R_c itself: a factor of 10^{16} for $E = 10^{-15}$ and 10^{11} for $E = 10^{-10}$. Negative values of small, this represents a huge fall in R_c itself: a factor of 10^{16} for $E = 10^{-15}$ and 10^{11} for $E = 10^{-10}$. Negative values of R_c correspond to convection driven by the imposed field. Such instabilities could 10^{11} for $E = 10^{-10}$. Negative values of R_c correspond to convection driven by the imposed field. Such instabilities could not persist indefinitely for a dynamo-driven field because they draw energy from the imposed imposed field. Such instabilities could not persist indefinitely for a dynamo-driven
field because they draw energy from the imposed field rather than the buoyancy
force, but they could be transients in a full dynamo calc field because they draw energy from the imposed field rather than the buoyancy
force, but they could be transients in a full dynamo calculation. The point $R_c = 0$
could, therefore, signify an upper bound on the strength o

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Figure 4. Large spatial scale and long time-scale magnetoconvection at $E = 10^{-5}$ with both Figure 4. Large spatial scale and long time-scale magnetoconvection at $E = 10^{-5}$ with both toroidal and poloidal magnetic fields imposed: $\Lambda = 10$, $\epsilon_{\rm P} = 0.017$ (see also table 1). Views (a) and (b) as in figure 1 Figure 4. Large spatial
toroidal and poloidal m
and (b) as in figure 1.

 $\frac{c}{c}$ and (b) as in figure 1.

corresponding drift rate of the rolls changes from positive (eastwards) to negative corresponding drift rate of the rolls changes from positive (eastwards) to negative (westwards) as the poloidal field increases. The profile of convection for $\epsilon_{\rm P} = 0.017$ is shown in figure 4: numerical values are gi corresponding drift rate of the rolls changes from positive
(westwards) as the poloidal field increases. The profile of cc
is shown in figure 4; numerical values are given in table 1.
These results show that linear magneto restwards) as the poloidal field increases. The profile of convection for $\epsilon_{\rm P} = 0.017$ shown in figure 4; numerical values are given in table 1.
These results show that linear magnetoconvection can be highly and criti

is shown in figure 4; numerical values are given in table 1.
These results show that linear magnetoconvection can be highly and critically
sensitive to small variations in poloidal field when the Ekman number is small. It
 These results show that linear magnetoconvection can be highly and critically
sensitive to small variations in poloidal field when the Ekman number is small. It
suggests that the amplitude and pattern of nonlinear convect sensitive to small variations in poloidal field when the Ekman number is small. It
suggests that the amplitude and pattern of nonlinear convection, which depends on
the difference $(R - R_c)$ (for example, (4.18)), will chang suggests that the amplitude and pattern of nonlinear convection, which depends on
the difference $(R - R_c)$ (for example, (4.18)), will change dramatically in response
to small variations in poloidal field. This in turn mean to small variations in poloidal field. This in turn means that the magnetic Reynolds
number $R_{\rm m}$ will change dramatically in response to small variations in poloidal field,
so that if the field were dynamo-generated r to small variations in poloidal field. This in turn means that the magnetic Reynolds
number $R_{\rm m}$ will change dramatically in response to small variations in poloidal field,
so that if the field were dynamo-generated r number $R_{\rm m}$ will change dramatically in response to small variations in poloidal field,
so that if the field were dynamo-generated rather than imposed we could expect small
perturbations in magnetic field to lead to m so that if the field were dynamo-generated rather than imposed we could expect small
perturbations in magnetic field to lead to much larger ones, or even a completely
different nonlinear solution: typical characteristics o perturbations in magnetic field to lead to much larger ones, or even a completely different nonlinear solution: typical characteristics of a highly unstable system. We have suggested, on the basis of this result, that a st

have suggested, on the basis of this result, that a steady convection-driven dynamo will not be stable if the dynamic contribution from the viscous term in equation (1.1) is neglected by enforcing (1.2) (Zhang & Gubbins 2 have suggested, on the basis of this result, that a steady c
will not be stable if the dynamic contribution from the visco
is neglected by enforcing (1.2) (Zhang & Gubbins 2000).
The second analysis is to integrate the Il not be stable if the dynamic contribution from the viscous term in equation (1.1) neglected by enforcing (1.2) (Zhang & Gubbins 2000).
The second analysis is to integrate the fully nonlinear equations (2.5)–(2.7) numer

The second analysis is to integrate the fully nonlinear equations $(2.5)-(2.7)$ numer-
ically for fixed Rayleigh number R and toroidal field with and without the poloidal field. It should be noted that the fully nonlinear magnetoconvection solution with ically for fixed Rayleigh number R and toroidal field with and without the poloidal field. It should be noted that the fully nonlinear magnetoconvection solution with both the toroidal and poloidal field is reported here field. It should be noted that the fully nonlinear magnetoconvection solution with
both the toroidal and poloidal field is reported here for the first time, but the lin-
ear stability calculations have been described in Z ear stability calculations have been described in Zhang $\&$ Gubbins (2000). A more detailed analysis of the nonlinear problem will be reported in a future paper. The inteear stability calculations have been described in Zhang & Gubbins (2000). A more
detailed analysis of the nonlinear problem will be reported in a future paper. The inte-
gration always starts from a random initial conditi , letailed analysis of the nonlinear problem will be reported in a future paper. The inter-
pration always starts from a random initial condition. For $R = 2.2 \times 10^4$, $E = 10^{-3}$,
 $\overline{P} = 0$, the final equilibrium solution gration always starts from a random initial condition. For $R = 2.2 \times 10^4$, $E = 10^{-3}$,
 $\epsilon_{\rm P} = 0$, the final equilibrium solution after a few magnetic diffusion times takes

the form of steadily drifting magnetoconvec $\epsilon_{\rm P} = 0$, the final equilibrium solution after a few magnetic diffusion times takes
the form of steadily drifting magnetoconvective waves with constant amplitude flow
and magnetic field; their phase speed is approxima the form of steadily drifting magnetoconvective waves with constant amplitude flow
and magnetic field; their phase speed is approximately predicted by linear analy-
sis (*C* in figure 3; see, for example, Zhang (1999)). T and magnetic field; their phase speed is approximately predic
sis (*C* in figure 3; see, for example, Zhang (1999)). The conve-
two-dimensional because of the Proudman-Taylor constraint.
We repeated the calculation with a $W(G)$ in figure 3; see, for example, Zhang (1999)). The convection is again nearly o-dimensional because of the Proudman–Taylor constraint.
We repeated the calculation with a weak poloidal field ($\epsilon_{\rm P} = 0.017$), keepin

We repeated the calculation with a weak poloidal field ($\epsilon_{\rm P} = 0.017$), keeping every-
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toroidal field (dot-dashed line), and poloidal magnetic field (dashed line) plotted as a function Figure 5. (a) Heat flux at the outer surface (solid line), the leading coefficient of axisymmetric toroidal field (dot-dashed line), and poloidal magnetic field (dashed line) plotted as a function of time for nonlinear ma toroidal field (dot-dashed line), and poloidal magnetic field (dashed line) plotted as a function
of time for nonlinear magnetoconvection with $R = 2.2 \times 10^4$, $A = 10$, $\epsilon_P = 0.017$ and $E = 10^{-3}$.
(b) Mean toroidal kinet (b) Mean toroidal kinetic energy (solid line) and mean poloidal kinetic energy (dashed line) plotted as a function of time for the same solution.

thing else the same. It changed the solution completely; the constant-amplitude travthing else the same. It changed the solution completely; the constant-amplitude travelling wave, which is steady in a corotating frame of reference, is replaced by vacillat-
ing magnetoconvection with large-amplitude varia thing else the same. It changed the solution completely; the constant-amplitude travelling wave, which is steady in a corotating frame of reference, is replaced by vacillating magnetoconvection with large-amplitude variati elling wave, which is steady in a corotating frame of reference, is replaced by vacillating magnetoconvection with large-amplitude variations in time. The time variation of the solution is displayed in figure 5, where the ing magnetoconvection with large-amplitude variations in time. The time variation
of the solution is displayed in figure 5, where the kinetic energy of mean toroidal
and poloidal convective motions, heat flux and dominant axisymmetric toroidal and poloidal magnetic fields are plotted as functions of time.

isymmetric toroidal and poloidal magnetic fields are plotted as functions of time.
6. The amplitude of fluid flow required to generate magnetic field

6. The amplitude of fluid flow required to generate magnetic field Further evidence of potential instability arises from kinematic studies of the geodynamo in which the fluid flow is fixed decoupling the induction equat Further evidence of potential instability arises from kinematic studies of the geodynamo in which the fluid flow is fixed, decoupling the induction equation (2.7), which Further evidence of potential instability arises from kinematic studies of the geodynamo in which the fluid flow is fixed, decoupling the induction equation (2.7), which may be solved for **B**. The problem is linear and, w namo in which the fluid flow is fixed, decoupling the induction equation (2.7), which
may be solved for **B**. The problem is linear and, when the flow is steady, presents
an eigenvalue problem for the critical magnetic Rey an eigenvalue problem for the critical magnetic Reynolds number R_m^c with **B** as the *Phil. Trans. R. Soc. Lond.* A (2000)

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Scale disparities in the geodynamo
eigenfunction. The solution for **B** can be very highly sensitive to the form of the
chosen flow **u** (Gubbins *et al.* 2000*a*), often to the extent that dynamo action fails eigenfunction. The solution for **B** can be very highly sensitive to the form of the chosen flow **u** (Gubbins *et al.* 2000*a*), often to the extent that dynamo action fails completely after a small change in flow. This re eigenfunction. The solution for **B** can be very highly sensitive to the form of the chosen flow **u** (Gubbins *et al.* 2000*a*), often to the extent that dynamo action fails completely after a small change in flow. This re chosen flow \bf{u} (Gubbins *et al.* 2000*a*), often to the extent that dynamo action fails completely after a small change in flow. This result parallels our findings for magnetoconvection to some extent, where the conv completely after a small change in flow. This result parallels our findings for magne-
toconvection to some extent, where the convective flow \bf{u} changes dramatically for
a small change in applied field \bf{B} . The n to convection to some extent, where the convective flow **u** changes dramatically for
a small change in applied field **B**. The nonlinear dynamo, in which the field is self-
generated, would, therefore, seem to be subject to a small change in applied field B . The nonlinear dynamo, in which the field is self-
generated, would, therefore, seem to be subject to a double instability, with small
variations in magnetic field producing large chang generated, would, therefore, seem to be subject to a double instability, with small variations in magnetic field producing large changes in flow, and small changes in flow producing large changes in the field.
The kinemati riations in magnetic field producing large changes in flow, and small changes in
w producing large changes in the field.
The kinematic dynamo problem is linear and, therefore, relatively easy to solve
merically, yet there

flow producing large changes in the field.
The kinematic dynamo problem is linear and, therefore, relatively easy to solve
numerically, yet there are very few examples of steady flow in a sphere generating
magnetic field, The kinematic dynamo problem is linear and, therefore, relatively easy to solve
numerically, yet there are very few examples of steady flow in a sphere generating
magnetic field, and almost no examples of steadily drifting μ magnetic field, and almost no examples of steadily drifting convection acting as a magnetic field, and almost no examples of steadily drifting convection acting as a dynamo. This suggests that time dependence of the flow is, perhaps, an important ingredient for dynamo action. To test the efficiency of s dynamo. This suggests that time dependence of the flow is, perhaps, an important
ingredient for dynamo action. To test the efficiency of steady flow in generating
magnetic field, Gubbins *et al.* (2000*a*) set up a two-par ingredient for dynamo action. To test the efficiency of steady flow in generating magnetic field, Gubbins *et al.* (2000*a*) set up a two-parameter class of fluid motions in a sphere that contained a small number of dynamos first found by Kumar & Roberts (1975) (see also Hutcheson & Gubbins 1994; Sarson & Gubbins 1996) and others found by Love & Gubbins (1996). Roberts (1975) (see also Hutcheson & Gubbins 1994; Sarson & Gubbins 1996) and

The flows comprise large-scale convective rolls with $m = 2$, differential rotation, $\ddot{\circ}$ and meridional circulation:

$$
\mathbf{u} = \epsilon_0 \mathbf{t}_1^0 + \epsilon_1 \mathbf{s}_2^0 + \epsilon_2 \mathbf{s}_2^{2c} + \epsilon_2 \mathbf{s}_2^{2s},\tag{6.1}
$$

 $\mathbf{u}=\epsilon_0\mathbf{t}_1^0+\epsilon_1\mathbf{s}_2^0+\epsilon_2\mathbf{s}_2^{2c}+\epsilon_2\mathbf{s}_2^{2s},$ where $\mathbf{t}_l^m,$ \mathbf{s}_l^m are toroidal and poloidal vector spherical harmonics,

$$
\mathbf{t}_{l}^{mc,s} = \nabla \times [t_{l}^{mc,s}(r)P_{l}^{m}(\sin\theta)(\cos,\sin(m\phi)\mathbf{e}_{r}], \qquad (6.2)
$$

$$
\mathbf{s}_{l}^{m\text{c},\text{s}} = \nabla \times \nabla \times [s_{l}^{m\text{c},\text{s}}(r)P_{l}^{m}(\cos\theta)[\cos,\sin](m\phi)\mathbf{e}_{r}],
$$
\n(6.3)

 $\mathbf{s}_l^{mc,s} = \nabla \times \nabla \times [s_l^{mc,s}(r)P_l^m(\cos\theta)[\cos,\sin](m\phi)\mathbf{e}_r],$ (6.3)
 \mathbf{e}_r is the unit vector in the r direction and superscripts 'c' and 's' denote cosine

and sine, respectively. The first harmonic in (6.1) represents e_r is the unit vector in the r direction and superscripts 'c' and 's' denote cosine
and sine, respectively. The first harmonic in (6.1) represents differential rotation,
the second represents meridional circulation, and e_r is the unit vector in the r direction and superscripts 'c' and 's' denote cosine
and sine, respectively. The first harmonic in (6.1) represents differential rotation,
the second represents meridional circulation, a and sine, respectively. The first harmonic in (6.1) represents differential rotation, the second represents meridional circulation, and the last two represent convective overturn. They provide what is thought to be the *AATHEMATICAL,*
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k ENGINEERING the second represents meridional circulation, and the last two represent convective
overturn. They provide what is thought to be the minimum complexity required to
generate a magnetic field possessing the basic features of overturn. They provide what is thought to be t
generate a magnetic field possessing the basic fe
and to mimic convection in a rotating sphere.
The scalar functions were chosen to give a u nerate a magnetic field possessing the basic features of the Earth's magnetic field
d to mimic convection in a rotating sphere.
The scalar functions were chosen to give a **u** that is differentiable at the origin,
d to be z

and to mimic convection in a rotating sphere.
The scalar functions were chosen to give a **u** that is and to be zero with zero stress on the outer boundary:

$$
t10(r) = r2(1 - r2),
$$

\n
$$
s20(r) = r6(1 - r2),
$$

\n
$$
s22(r) = r6(1 - r2)3,
$$

\n
$$
s22(r) = r4(1 - r2)2 cos(pπr),
$$

\n
$$
s22s(r) = r4(1 - r2)2 sin(pπr).
$$
\n(6.4)

THE ROYAL $s_2^{2s}(r) = r^4(1 - r^2)^2 \sin(p\pi r)$.
The flows are parametrized by the fraction of energy in the differential rotation
(*D*) meridional circulation (*M*) and convection (*C* = 1 - |*D*| - |*M*|) Solutions The flows are parametrized by the fraction of energy in the differential rotation (D) , meridional circulation (M) , and convection $(C = 1 - |D| - |M|)$. Solutions to the induction equation with this flow separate into four sy The flows are parametrized by the fraction of energy in the differential rotation (D) , meridional circulation (M) , and convection $(C = 1 - |D| - |M|)$. Solutions to the induction equation with this flow separate into four sy Ω (*D*), meridional circulation (*M*), and convection (*C* = 1 - |*D*| - |*M*|). Solutions to the induction equation with this flow separate into four symmetries, the dipole and quadrupole solutions referred to in § 3 to the induction equation with this flow separate into four symmetries, the dipole and quadrupole solutions referred to in $\S 3$ for convection, and two further solutions characterized by odd azimuthal wavenumbers (the so and quadrupole solutions referred to in $\S 3$ for convection, and two further solutions aracterized by odd azimuthal wavenumbers (the so-called equatorial dipole and
adrupole solutions).
Gubbins *et al.* (2000*a*) found that only 36% of the flows defined by the two param-
prs *D* and *M* generated magnetic fi

quadrupole solutions).
Gubbins *et al.* (2000*a*) found that only 36% of the flows defined by the two parameters D and M generated magnetic fields with dipole symmetry, and that these eters D and M generated magnetic fields with dipole symmetry, and that these $Phil.$ *Trans. R. Soc. Lond.* A (2000)

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914 \blacksquare \blacksquare flows were confined to seven separate zones in (D, M) parameter space. Surprisingly, flows in different zones often generate magnetic fields with similar morphologies. The boundaries of these zones are characterized by sm flows were confined to seven separate zones in (D, M) parameter space. Surprisingly,
flows in different zones often generate magnetic fields with similar morphologies. The
boundaries of these zones are characterized by sm boundaries of these zones are characterized by small-scale magnetic fields and large critical magnetic Reynolds numbers, where the flow either concentrates the field into boundaries of these zones are characterized by small-scale magnetic fields and large
critical magnetic Reynolds numbers, where the flow either concentrates the field into
very narrow bands producing diffusion, or expels it critical magnetic Reynolds numbers, where the flow either concentrates the field into
very narrow bands producing diffusion, or expels it from the sphere with consequent
loss of dynamo generation. Within any zone, it is po

very narrow bands producing diffusion, or expels it from the sphere with consequent
loss of dynamo generation. Within any zone, it is possible to change the flow param-
eters considerably without changing the qualitative n the are also places where a tiny change in solution at the dynamo action, yet
there are also places where a tiny change in flow will change the solution completely.
In a second study. Gubbins *et al.* (2000b) explored oth In a second study, Gubbins *et al.* (2000*b*) explored other symmetries and found at nearly half of the flows generated fields with at least one symmetry, some could

there are also places where a tiny change in flow will change the solution completely.
In a second study, Gubbins et al . (2000 b) explored other symmetries and found that nearly half of the flows generated fields with In a second study, Gubbins *et al.* (2000*b*) explored other symmetries and found that nearly half of the flows generated fields with at least one symmetry, some could generate two or more symmetries with different R_m^c that nearly half of the flows generated fields with at least one symmetry, some could
generate two or more symmetries with different R_m^c , and some could produce two
different symmetries with the same R_m^c . This last generate two or more symmetries with different R_m^c , and some could produce two
different symmetries with the same R_m^c . This last case delineates a boundary in
parameter space that separates physically realizable sol different symmetries with the same R_m^c . This last case delineates a boundary in parameter space that separates physically realizable solutions: on this boundary, an infinitesimal change in flow parameters would change parameter space that separates physically realizable solutions: on this boundary, an infinitesimal change in flow parameters would change the entire nature of the generated field. Flows with $D > 0$, which correspond to pr an infinitesimal change in flow parameters would change the entire nature of the generated field. Flows with $D > 0$, which correspond to primarily westward flow
at the surface of the sphere, generated axial dipole field solutions that were almost
exclusively steady. A very small, but perhaps significa produced oscillatory solutions.
For some flows, the critical magnetic Reynolds number was found to depend very clusively steady. A very small, but perhaps significant, proportion of the flows
oduced oscillatory solutions.
For some flows, the critical magnetic Reynolds number was found to depend very
nsitively on changes in the flow

produced oscillatory solutions.
For some flows, the critical magnetic Reynolds number was found to depend very
sensitively on changes in the flow: by a factor of 3 with a 0.1% change in flow and
with the appearance of asy For some flows, the critical magnetic Reynolds number was found to depend very sensitively on changes in the flow: by a factor of 3 with a 0.1% change in flow and with the appearance of asymptoting to infinity with a f sensitively on changes in the flow: by a factor of 3 with a 0.1% change in flow and
with the appearance of asymptoting to infinity with a finite change in one of the
flow parameters (D or M). These rapid changes occur with the appearance of asymptoting to infinity with a finite change in one of the flow parameters (D or M). These rapid changes occurred on the boundaries of zones of dynamo action, and within zones where the steady s flow parameters (*D* or *M*). These rapid changes occurred on the boundaries of zones of dynamo action, and within zones where the steady solution is replaced with an oscillatory solution for a very small interval in $M:$ of dynamo acti

<u>oscillatory</u> solut
 et al. 2000*a*).

These kinema oscillatory solution for a very small interval in $M: -0.010 < M < -0.002$ (Gubbins *et al.* 2000*a*).
These kinematic studies may have implications for the full nonlinear dynamo prob-

These kinematic studies may have implications for the full nonlinear dynamo prob-
lem. A time-dependent solution will explore a space of fluid velocities. The obser-
vation that the Earth has possessed a non-oscillatory ma lem. A time-dependent solution will explore a space of fluid velocities. The obserlem. A time-dependent solution will explore a space of fluid velocities. The observation that the Earth has possessed a non-oscillatory magnetic field with dipole symmetry for most of its history strongly suggests that th **MATHEMATICAL,
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& ENGINEERING
SCIENCES** vation that the Earth has possessed a non-oscillatory magnetic field with dipole
symmetry for most of its history strongly suggests that the flow has a permanent
characteristic like the $D > 0$ 'westward drift' of this mod symmetry for most of its history strongly suggests that the flow has a permanent
characteristic like the $D > 0$ 'westward drift' of this model. Another observation,
that it reverses occasionally, suggests that the flow ma characteristic like the $D > 0$ 'westward drift' of this model. Another observation, that it reverses occasionally, suggests that the flow may occasionally range into a region where another symmetry is generated, or an osc region where another symmetry is generated, or an oscillatory solution is preferred.
The third possibility is that the flow takes a form that cannot generate magnetic region where another symmetry is generated, or an oscillatory solution is preferred.
The third possibility is that the flow takes a form that cannot generate magnetic
field, or $R_{\rm m}^{\rm c}$ increases dramatically, making The third possibility is that the flow takes a form that cannot generate magnetic field, or $R_{\rm m}^{\rm c}$ increases dramatically, making dynamo action inefficient. This strong dependence of the dynamo action on the precis field, or $R_{\rm m}^{\rm c}$ increases dramatically, making
dependence of the dynamo action on the pre
of instability in the full nonlinear dynamo. % of instability in the full nonlinear dynamo.
 7. Discussion

This paper was prompted by two developments: the theoretical result that strong- This paper was prompted by two developments: the theoretical result that strong-
field dynamo models often collapse and lead to non-magnetic convection, and the
observation that the geomagnetic field also appears to have s This paper was prompted by two developments: the theoretical result that strong-
field dynamo models often collapse and lead to non-magnetic convection, and the
observation that the geomagnetic field also appears to have s field dynamo models often collapse and lead to non-magnetic convection, and the observation that the geomagnetic field also appears to have suffered frequent collapses in the form of excursions: large departures from the a observation that the geomagnetic field also appears to have suffered frequent collapses
in the form of excursions: large departures from the axial dipole form and order-
of-magnitude falls in strength. Taken together, thes in the form of excursions: large departures from the axial dipole form and order-
of-magnitude falls in strength. Taken together, these results make it important to
understand the stability of the strong-field dynamo in th of-magnitude falls in strength. Taken together, these results make it impounderstand the stability of the strong-field dynamo in the small-Ekman-numb
the only model that can explain the geomagnetic field in its present for understand the stability of the strong-field dynamo in the small-Ekman-number limit, the only model that can explain the geomagnetic field in its present form.
Instability of the strong-field dynamo at small Ekman number p

numerical difficulties. We have two choices: either assume the strong-field dynamo is

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stable and compute solutions at modest Rayleigh number and small Ekman number (but recent calculations suggest this does not work), or resolve the weak-field solustable and compute solutions at modest Rayleigh number and small Ekman number
(but recent calculations suggest this does not work), or resolve the weak-field solu-
tion that may develop if the dynamo collapses. The second (but recent calculations suggest this does not work), or resolve the weak-field solution that may develop if the dynamo collapses. The second alternative is safe but impossible for small Ekman number. Our estimates of cri tion that may develop if the dynamo collapses. The second alternative is safe but
impossible for small Ekman number. Our estimates of critical Rayleigh numbers in
 $\S\S 3$ and 4 differ by a factor of $7E^{-1/3}$ (cf. (3.5) an

impossible for small Ekman number. Our estimates of critical Rayleigh numbers in $\S\S 3$ and 4 differ by a factor of $7E^{-1/3}$ (cf. (3.5) and (4.13)), or 7×10^5 or 3×10^4 depending on whether molecular or eddy diff $\S\S 3$ and 4 differ by a factor of $7E^{-1/3}$ (cf. (3.5) and (4.13)), or 7×10^5 or 3×10^4
depending on whether molecular or eddy diffusivities are used. Starting a strong-field
simulation with a Rayleigh number les depending on whether molecular or eddy diffusivities are used. Starting a strong-field simulation with a Rayleigh number less than several thousand times critical could lead to the convection shutting down completely if t simulation with a Rayleigh number less than several thousand times critical could
lead to the convection shutting down completely if the field dropped and R_c rose
to exceed R. Even with a high value of R, we would have lead to the convection shutting down completely if the field dropped and R_c rose
to exceed R . Even with a high value of R , we would have to resolve small length-
scales ($m = 70000$ or 1600) and, even more seriously, to exceed R. Even with a high value of R, we would have to resolve s
scales $(m = 70000 \text{ or } 1600)$ and, even more seriously, very short time-sc
numerical calculation will remain impossible in the foreseeable future.
Our ma ales $(m = 70000$ or 1600) and, even more seriously, very short time-scales. Such a merical calculation will remain impossible in the foreseeable future.
Our main hope for understanding the geodynamo must rely on extrapolat

numerical calculation will remain impossible in the foreseeable future.

Our main hope for understanding the geodynamo must rely on extrapolations of

convection-driven dynamo solutions to small Ekman numbers, similar to Our main hope for understanding the geodynamo must rely on extrapolations of convection-driven dynamo solutions to small Ekman numbers, similar to those in (3.6) and (4.12). We do not yet understand this extrapolation beca convection-driven dynamo solutions to small Ekman numbers, similar to those in (3.6) and (4.12) . We do not yet understand this extrapolation because we do not understand the implications for magnetoconvection when the (3.6) and (4.12) . We do not yet understand this extrapolation because we do not understand the implications for magnetoconvection when the magnetic field is self-
generated: the dynamo instability and its effect on co understand the implications for magnetoconvection when the magnetic field is self-
generated: the dynamo instability and its effect on convection has not been studied
simply because most convective flows fail to generate a generated: the dynamo instability and its effect on convection has not been studied
simply because most convective flows fail to generate a magnetic field. Furthermore,
the numerical calculation is intractable even at mod simply because most convective flows fail to generate a magnetic field. Furthermore,
the numerical calculation is intractable even at modest values of the Ekman number.
At $E = 10^{-6}$, for example, we would have to resolve the numerical calculation is intractable even at modest values of the Ekman number.
At $E = 10^{-6}$, for example, we would have to resolve wavenumbers out to $m = 200$
(double the expected wavenumber in order to include all At $E = 10^{-6}$, for example, we would have to resolve wavenumbers out to $m = 200$ (double the expected wavenumber in order to include all primary convective modes) with a time-step $\Delta t < 10^{-6}$ diffusion times in order to (double the expected wavenumber in order to include all primary convective modes) with a time-step $\Delta t < 10^{-6}$ diffusion times in order to resolve the drift frequency of the rolls (10^{-4}) . The disparity of scales shown with a time-step $\Delta t < 10^{-6}$ diffusion times in order to resolve the drift frequency of the rolls (10⁻⁴). The disparity of scales shown in table 1 explains why all the recent geodynamo models (see, for example, Sarson the rolls (10^{-4}) . The disparity of scales shown in table 1 explains why all the recent geodynamo models (see, for example, Sarson *et al.* 1998; Olson *et al.* 1999; Katayama *et al.* 1999) have difficulty in reaching geodynamo models (see, for example, Sarson *et al.* 1998; Olson *et al.* 1999; *et al.* 1999) have difficulty in reaching $E < 10^{-4}$. It is evident that $E \ge \text{sufficiently small}$ for the results to be extrapolated to the Earth's core. al. 1999) have difficulty in reaching $E < 10^{-4}$. It is evident that $E \ge 10^{-4}$ is not
fficiently small for the results to be extrapolated to the Earth's core.
A simple way of removing the scale disparities is to introdu

sufficiently small for the results to be extrapolated to the Earth's core.
A simple way of removing the scale disparities is to introduce hyperviscosity, which
is related to the idea of local turbulence and cascades in atm A simple way of removing the scale disparities is to introduce hyperviscosity, which
is related to the idea of local turbulence and cascades in atmospheric dynamics. There
are two objections to the application of hypervisc is related to the idea of local turbulence and cascades in atmospheric dynamics. There are two objections to the application of hyperviscosity to the Earth's core. The first is the lack of an established turbulent MHD theo are two objections to the application of hyperviscosity to the Earth's core. The first
is the lack of an established turbulent MHD theory. The second, and more important,
objection is that the dynamics is fundamentally dif is the lack of an established turbulent MHD theory. The second, and more important,
objection is that the dynamics is fundamentally different from that of the atmosphere.
The dynamo problem operates on such a long time-sc objection is that the dynamics is fundamentally different from that of the atmosphere.
The dynamo problem operates on such a long time-scale that the effect of the inertial
term in (2.1), $[\mathbf{u} \cdot \nabla \mathbf{u}]$, is, dynamic The dynamo problem operates on such a long time-scale that the effect of the inertial
term in (2.1), $[\mathbf{u} \cdot \nabla \mathbf{u}]$, is, dynamically, of secondary importance. Regardless of the
amplitude of convection, the governing term in (2.1), $[\mathbf{u} \cdot \nabla \mathbf{u}]$, is, dynamically, of secondary importance. Regardless of the amplitude of convection, the governing equation of motion is effectively linear when and where the generated magnetic field and where the generated magnetic field is weak. In order that convection takes place, and where the generated magnetic field is weak. In order that convection takes place,
the scale of motion must be sufficiently small, as clearly shown by equation (3.1).
The dynamic role of viscosity, as explained by Chan the scale of motion must be sufficiently small, as clearly shown by equation (3.1).
The dynamic role of viscosity, as explained by Chandrasekhar (1961) (see also Zhang & Busse 1998), is to provide the frictional force nece $\geq k$ Busse 1998), is to provide the frictional force necessary to offset the Coriolis force to allow convection. In atmospheric dynamics, or convection in the form of thermal-& Busse 1998), is to provide the frictional force necessary to offset the Coriolis force
to allow convection. In atmospheric dynamics, or convection in the form of thermal-
inertial waves, the inertial term $\mathbf{u} \cdot \nabla \$ to allow convection. In atmospheric dynamics, or convection in the form of the
inertial waves, the inertial term $\mathbf{u} \cdot \nabla \mathbf{u}$ sets up the turbulent energy cascade
viscosity plays its conventional role of dissipati existed waves, the inertial term $\mathbf{u} \cdot \nabla \mathbf{u}$ sets up the turbulent energy cascade, and scosity plays its conventional role of dissipating the smallest length-scales.
Our ideas about the instability are based on m

viscosity plays its conventional role of dissipating the smallest length-scales.
Our ideas about the instability are based on magnetoconvection with a constant applied field and kinematic dynamos with constant velocity; we Our ideas about the instability are based on magnetoconvection with a constant
applied field and kinematic dynamos with constant velocity; we do not know the
implications of time-varying applied \bf{B} or \bf{u} for eit applied field and kinematic dynamos with constant velocity; we do not know the
implications of time-varying applied \bf{B} or \bf{u} for either case. Indeed, the comparative
ease of finding time-dependent dynamos over t implications of time-varying applied \bf{B} or \bf{u} for either case. Indeed, the comparative
ease of finding time-dependent dynamos over those with steady flow suggests that
time dependence is an important factor, and ease of finding time-dependent dynamos over those with steady flow suggests that
time dependence is an important factor, and a time-varying magnetic field may
change the nature of magnetoconvection. Both of these problems me dependence is an important factor, and a time-varying magnetic field may
ange the nature of magnetoconvection. Both of these problems are under study.
How do we resolve these conflicts of scale? One possible scenario is

change the nature of magnetoconvection. Both of these problems are under study.
How do we resolve these conflicts of scale? One possible scenario is that a dynamo
is neither strong nor weak: it swings between a strong-fiel *Phil. Trans. R. Soc. Lond.* A (2000)

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(4.13) and a weak-field dynamo with scaling (3.5). Localized small-scale convection (4.13) and a weak-field dynamo with scaling (3.5) . Localized small-scale convection with a weak field is ineffective in transporting heat, making it unstable to the dynamo instability, while a strong-field dynamo withou (4.13) and a weak-field dynamo with scaling (3.5). Localized small-scale convection
with a weak field is ineffective in transporting heat, making it unstable to the dynamo
instability, while a strong-field dynamo without instability, while a strong-field dynamo without viscous effects appears to be unstable because of the high variability of R_c with magnetic field and of R_m^c with fluid instability, while a strong-field dynamo without viscous effects appears to be unstable because of the high variability of R_c with magnetic field and of R_m^c with fluid flow. Such vacillation between strong- and weakble because of the high variability of R_c with magnetic field and of R_m^c with fluid flow. Such vacillation between strong- and weak-field states could provide a natural explanation for the repeated falls in field str flow. S
ral exp
field.
Holle ral explanation for the repeated falls in field strength observed in the geomagnetic
field.
Hollerbach (1997) has already outlined a similar idea, the existence of an inter-

mediate dynamo state (the semi-Taylor state) based on an $\alpha\omega$ dynamo model in a Hollerbach (1997) has already outlined a similar idea, the existence of an inter-
mediate dynamo state (the semi-Taylor state) based on an $\alpha\omega$ dynamo model in a
rotating spherical system, which is between the strong- a mediate dynamo state (the semi-Taylor state) based on an $\alpha\omega$ dynamo model in a rotating spherical system, which is between the strong- and weak-field states. He suggested that the semi-Taylor state, which describes a t rotating spherical system, which is between the strong- and weak-field states. He
suggested that the semi-Taylor state, which describes a temporary departure of the
dynamo solutions from the strong-field (Taylor) state and suggested that the semi-Taylor state, which describes a temporary departure of the dynamo solutions from the strong-field (Taylor) state and which has a field strength intermediate between the strong- and weak-field dynamo dynamo solutions from the strong-field (Taylor) state and which has a field strength
intermediate between the strong- and weak-field dynamos, might be relevant to geo-
magnetic excursions. The weak-field and semi-Taylor st intermediate between the strong- and weak-field dynamos, might be relevant to geo-
magnetic excursions. The weak-field and semi-Taylor states will be very difficult
to distinguish from palaeomagnetic observations because i magnetic excursions. The weak-field and semi-Taylor states will be very difficult
to distinguish from palaeomagnetic observations because it involves differentiating
between a drop in field intensity of perhaps 100 and a d to distinguish from palaeomagnetic observations because it involves differentiating temporal averaging and background noise prevent us from measuring any signal that Although palaeomagnetism suggests the fall is by a factor of 5–10 during reversals,
temporal averaging and background noise prevent us from measuring any signal that
is significantly smaller than this. Furthermore, the dyn temporal averaging and background nois
is significantly smaller than this. Furthe
toroidal field, which is not observable.
Finally, consider now how to apply sp significantly smaller than this. Furthermore, the dynamics depends mainly on the roidal field, which is not observable.
Finally, consider now how to apply such a model to the Earth. The present state the core and geomagnet

toroidal field, which is not observable.
Finally, consider now how to apply such a model to the Earth. The present state
of the core and geomagnetic field corresponds to a strong-field dynamo, in which the Finally, consider now how to apply such a model to the Earth. The present state
of the core and geomagnetic field corresponds to a strong-field dynamo, in which the
primary force balance is between Coriolis and Lorentz for of the core and geomagnetic field corresponds to a strong-field dynamo, in which the
primary force balance is between Coriolis and Lorentz forces. Three quantities should
have roughly the right order of magnitude: the conv primary force balance is between Coriolis and Lorentz forces. Three quantities should
have roughly the right order of magnitude: the convected heat flux; the ohmic heating
associated with the generated field; and the magne have roughly the right order of magnitude: the convected heat flux; the ohmic heating associated with the generated field; and the magnetic Reynolds number. There is no problem meeting these three requirements with a stron associated with the generated field; and the magnetic Reynolds number. There is no

flux extracted through the core-mantle boundary as a result of mantle convection, Core convection is determined not by an applied Rayleigh number but by the heat
flux extracted through the core-mantle boundary as a result of mantle convection,
which remains constant on the time-scales of interest for th flux extracted through the core-mantle boundary as a result of mantle convection,
which remains constant on the time-scales of interest for the geodynamo. The heat
flux must be greater than that conducted down the adiabati which remains constant on the time-scales of interest for the geodynamo. The heat
flux must be greater than that conducted down the adiabatic temperature gradient
by the molecular value of the thermal diffusivity; modern flux must be greater than that conducted down the adiabatic temperature gradient
by the molecular value of the thermal diffusivity; modern estimates place this at
about 10^{12} W (Labrosse *et al.* 1997). Any additional by the molecule
about 10^{12} W
fluid motion.
Sustaining the out 10^{12} W (Labrosse *et al.* 1997). Any additional heat must be convected by idention.
Sustaining the geomagnetic field by dynamo action driven by thermal convection quires a heat throughout equal to the ohmic heatin

fluid motion.
Sustaining the geomagnetic field by dynamo action driven by thermal convection
requires a heat throughput equal to the ohmic heating multiplied by a thermody-
namic efficiency factor of about 10 (Backus 1975 Sustaining the geomagnetic field by dynamo action driven by thermal convection
requires a heat throughput equal to the ohmic heating multiplied by a thermody-
namic efficiency factor of about 10 (Backus 1975; Hewitt *et al* requires a heat throughput equal to the ohmic heating multiplied by a thermody-
namic efficiency factor of about 10 (Backus 1975; Hewitt *et al.* 1975; Gubbins 1977).
This result cannot be determined directly from the Bou namic efficiency factor of about 10 (Backus 1975; Hewitt *et al.* 1975; Gubbins 1977).
This result cannot be determined directly from the Boussinesq approximation, which
implicitly assumes that the ohmic heating is neglig This result cannot be determined directly from the Boussinesq approximation, which
implicitly assumes that the ohmic heating is negligible; it is a more fundamental
result and, therefore, a good guide to the heat flux we s Implicitly assumes that the ohmic heating is negligible; it is a more fundamental
result and, therefore, a good guide to the heat flux we should expect. If using turbu-
lent diffusivities, we should also consider enhanced result and, therefore, a good guide to the heat flux we should expect. If using turbu-
lent diffusivities, we should also consider enhanced viscous and thermally diffusive
contributions to the entropy, increasing the heat lent diffusivities, we should also consider enhanced viscous and thermally diffusive
contributions to the entropy, increasing the heat requirements still further. Com-
positional convection can reduce the requirements some contributions to the entropy, increasing the heat requirements still further. Compositional convection can reduce the requirements somewhat (Gubbins *et al.* 1979; Loper 1978). These differences are unimportant for the pr positional convection can reduce the requirements somewhat (Gubbins *et al.* 1979; Loper 1978). These differences are unimportant for the present discussion: the essential requirement is that core convection provides some throughput. If requirement is that core convection provides something like $10^{10} - 10^{12}$ W of heat
roughput.
The estimate (4.15) for the heat flux from strong-field magnetoconvection yields
ly 10^8 W. This is too small: it would

The estimate (4.15) for the heat flux from strong-field magnetoconvection yields only $10⁸$ W. This is too small: it would indeed be amazing if the mantle were to impose a heat flux equal to that required to maintain the adiabat (10^{11} W) plus

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just an additional 0.1% for the convection! Furthermore, there is a contradiction just an additional 0.1% for the convection! Furthermore, there is a contradiction
with the ohmic heating associated with the applied field in the calculation, which
is significantly greater than 10^8 W. Most of the e just an additional 0.1% for the convection! Furthermore, there is a contradiction
with the ohmic heating associated with the applied field in the calculation, which
is significantly greater than 10^8 W. Most of the e is significantly greater than 10^8 W. Most of the energy lost to ohmic heating comes
from the (unspecified) source of the applied field; the convection could not, therefore,
generate the imposed field by dynamo action, is significantly greater than 10^8 W. Most of the energy lost to ohmic heating comes
from the (unspecified) source of the applied field; the convection could not, therefore,
generate the imposed field by dynamo action, from the (unspecified) source of the applied field; the generate the imposed field by dynamo action, despit to give a respectable magnetic Reynolds number.
In a steady dynamo, a simple integral of the ine

In a steady dynamo, a simple integral of the induction equation shows that the to give a respectable magnetic Reynolds number.
In a steady dynamo, a simple integral of the induction equation shows that the
work done against Lorentz forces is equal to the ohmic heating; in a non-steady
dynamo, this ba In a steady dynamo, a simple integral of the induction equation shows that the work done against Lorentz forces is equal to the ohmic heating; in a non-steady dynamo, this balance must be maintained in the time average. In work done against Lorentz forces is equal to the ohmic heating; in a non-steady dynamo, this balance must be maintained in the time average. In the magnetoconvection calculation there is an imbalance that will lead to deca dynamo, this balance must be maintained in the time average. In the magnetoconvection calculation there is an imbalance that will lead to decay of the field if the extra source is removed. To model the Earth as it is toda extra source is removed. To model the Earth as it is today we must maintain the same typical flow speed, \mathcal{U} , while increasing the Lorentz force. This means a different force extra source is removed. To model the Earth as it is today we must maintain the same
typical flow speed, U , while increasing the Lorentz force. This means a different force
balance, and probably a different Rayleigh num typical flow speed, U , while increasing the Lorentz force. This means a different force balance, and probably a different Rayleigh number. We should not therefore rely on (4.18) as a reliable estimator for the Rayleig balance, and probably a different Rayleigh number. We should not the (4.18) as a reliable estimator for the Rayleigh number, although its E , crucial to the main argument of this paper, remains unaffected.
Now, suppose t (4.18) as a reliable estimator for the Rayleigh number, although its dependence on E , crucial to the main argument of this paper, remains unaffected.
Now, suppose the field collapses and a weak-field regime, similar t

convection, is established. We imagine this weak-field regime corresponds with onset of an excursion, or one of the dips in relative intensity seen in many sediment records Now, suppose the field collapses and a weak-field regime, similar to non-magnetic convection, is established. We imagine this weak-field regime corresponds with onset of an excursion, or one of the dips in relative intensi convection, is established. We imagine this weak-field regime corresponds with onset
of an excursion, or one of the dips in relative intensity seen in many sediment records
(see, for example, Channel, this issue). The weak of an excursion, or one of the dips in relative intensity seen in many sediment records
(see, for example, Channel, this issue). The weak-field dynamo does not scale to the
Earth easily. We have argued that small-scale co Earth easily. We have argued that small-scale convection is inefficient at transporting heat, a view sustained by (3.9) , which gives only 10^7 W for typical present-day core flow speeds. However, we should not restrict U to present-day values when the correct is in such a dramatically different regime; we should instead adopt the correct heat flux and estimate the resulting flow, which vari flow speeds. However, we should not restrict U to present-day values when the core
is in such a dramatically different regime; we should instead adopt the correct heat
flux and estimate the resulting flow, which varies is in such a dramatically different regime; we should instead adopt the correct heat flux and estimate the resulting flow, which varies as the square root of the heat flux according to (4.15) and (4.18). Raising the heat flux and estimate the resulting flow, which
according to (4.15) and (4.18) . Raising the
speed by a factor of 100, to *ca*. 5 mm s⁻¹.
Small-scale motions are also inefficient a cording to (4.15) and (4.18). Raising the heat flux to 10^{11} W increases the flow
eed by a factor of 100, to ca. 5 mm s⁻¹.
Small-scale motions are also inefficient at generating magnetic field. The typical
ro-scale m

speed by a factor of 100, to ca. 5 mm s⁻¹.
Small-scale motions are also inefficient at generating magnetic field. The typical
two-scale mechanism (see, for example, Busse 1975) generates field in a two-stage
process in Small-scale motions are also inefficient at
two-scale mechanism (see, for example, Bu
process in which (1) small-scale field \mathbf{b}^0 is i
large-scale field is induced by the average of Small-scale motions are also inefficient at generating magnetic field. The typical **MATHEMATICAL,
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process in which (1) small-scale field \mathbf{b}° is induced from large scale field $\bar{\mathbf{B}}$ and (2)
large-scale field is induced by th process in which (1) small-scale field \mathbf{b}° is induced from large scale field **B** and (2) process in which (1) small-scale field \mathbf{b}^0 is induced from large scale field \mathbf{B} and (2) large-scale field is induced by the average of the action of small-scale flow on small-scale field, $\nabla \times (\mathbf{u}^0 \times \mathbf{$ large-scale field is induced by the average of the action
scale field, $\nabla \times (\mathbf{u}^0 \times \mathbf{b}^0)$. Balancing terms in the dimen
equation, where the large length-scale is $O(1)$, gives

$$
R_{\rm m} = O(l^{-1/2}) \approx 100m^{1/2} = O(E^{-1/6}),\tag{7.1}
$$

where the constant factor appearing in (7.1) is justified from calculations for dynamos where the constant factor appearing in (7.1) is justified from calculations for dynamos
generating large-scale $(m = 1)$ magnetic fields, which have $R_{\rm m}^{\rm c} \approx 100$. For $E = 10^{-15}$,
(7.1) gives $R_{\rm m} \approx 3 \times 10^4$ and where the constant factor appearing in (7.1) is justified from calculations for dynamos
generating large-scale $(m = 1)$ magnetic fields, which have $R_{\rm m}^{\rm c} \approx 100$. For $E = 10^{-15}$,
(7.1) gives $R_{\rm m} \approx 3 \times 10^4$, and generating large-scale $(m = 1)$ magnetic fields, which have $R_{\rm m}^{\rm c} \approx 100$. For $E = 10^{-15}$,
(7.1) gives $R_{\rm m} \approx 3 \times 10^4$, and for $E = 10^{-10}$ it gives $R_{\rm m} \approx 3 \times 10^4$, and for $E = 10^{-10}$
it gives $R_{\rm m} \approx 40$ (7.1) gives $R_m \approx 3 \times 10^4$, and for $E = 10^{-10}$ it gives $R_m \approx 3 \times 10^4$, and for $E = 10^{-10}$ it gives $R_m \approx 4000$. The corresponding flow speeds in the core are 0.3 m s⁻¹ and 4 mm s⁻¹, respectively. Thus, we must ha it gives $R_m \approx 4000$. The corresp
4 mm s⁻¹, respectively. Thus, we
dynamo when the field is weak.
There remains the problem of sa nm s⁻¹, respectively. Thus, we must have fast core flow to continue to maintain a
mamo when the field is weak.
There remains the problem of satisfying the ohmic heating associated with a small-
ale field. Using the same

dynamo when the field is weak.
There remains the problem of satisfying the ohmic heating associated with a small-
scale field. Using the same two-scale model as above, the ohmic heating $(\nabla \times \mathbf{b})^2$
scales as $R^2 (\nabla$ $(0)^2$ There remains the problem of satisfying the ohmic heating associated with a small-
scale field. Using the same two-scale model as above, the ohmic heating $(\nabla \times \mathbf{b})^2$
scales as $R_{\text{m}}^2 (\nabla \times \mathbf{B})^2$, an increase $\frac{2}{m} (\nabla \times \vec{\mathbf{B}})^2$, a scale field. Using the same two-scale model as above, the ohmic heating $(\nabla \times \mathbf{b})^2$
scales as $R_m^2 (\nabla \times \mathbf{B})^2$, an increase of a factor of 10^7 for molecular diffusivities and
1600 for turbulent values. $\mathbf{\bar{$ 1600 for turbulent values. **B** is taken to be one-tenth the size of the present core field in this regime. The molecular value is hard to reconcile: over 10^{13} W. However, the 1600 for turbulent values. **B** is taken to be one-tenth the s
in this regime. The molecular value is hard to reconcile:
ohmic heating for eddy diffusivities lies within bounds.
In summary the task of obtaining an Earth-lik this regime. The molecular value is hard to reconcile: over 10^{13} W. However, the mic heating for eddy diffusivities lies within bounds.
In summary, the task of obtaining an Earth-like and self-sustaining numerical geo

In summary, the task of obtaining an Earth-like and self-sustaining numerical geodynamo model remains a major challenge because of the scale disparities associated

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918 \blacksquare \blacksquare ficulties in modelling a convection-driven geodynamo but is also the key feature of

the dynamics of the Earth's fluid core.
D.G. was supported by NERC grant GR3/9741. K.Z. is supported by an NERC and a PPARC the dynamics of the Earth's fluid core.
D.G. was supported by NERC grant GR3/9741. K.Z. is supported by an NERC and a PPARC
grant. We thank C. A. Jones and B. Hollerbach for useful discussions D.G. was supported by NERC grant GR3/9741. K.Z. is supported by an grant. We thank C. A. Jones and R. Hollerbach for useful discussions.

References
Backus, G. E. 1975 Gross thermodynamic heat engines in deep interior of Earth. *Proc. Natn.*
Acad Sci. USA 72, 1555–1558 *Acad. Sci. USA* 72, 1555–1558.
Acad. Sci. USA 72, 1555–1558. Backus, G. E. 1975 Gross thermodynamic heat engines in deep interior of Earth. *Proc. N*
Acad. Sci. USA **72**, 1555–1558.
Busse, F. H. 1970 Thermal instabilities in rotating systems. *J. Fluid Mech.* 44, 444–460.
Busse, F

Acad. Sci. USA 72, 1555–1558.
Busse, F. H. 1970 Thermal instabilities in rotating systems. *J. Fluid Mech.* 44, 444–460
Busse, F. H. 1975 A model of the geodynamo. *Geophys. J. R. Astr. Soc.* 42, 437–459.
Busse, F. H. 19

Busse, F. H. 1970 Thermal instabilities in rotating systems. *J. Fluid Mech.* 44, 444–460.
Busse, F. H. 1975 A model of the geodynamo. *Geophys. J. R. Astr. Soc.* 42, 437–459.
Busse, F. H. 1978 Magnetohydrodynamics of the Busse, F. H. 1978 Magnetohydrodynamics of the Earth's dynamo. A. Rev. Fluid Mech
435–462.
Chandrasekhar, S. 1961 *Hydrodynamic and hydromagnetic stability*. Oxford: Clarendon.
Form D. B. 1979 Thermal and magnetic instabili

- Fearn, D. R. 1979 Thermal and magnetic instabilities in a rapidly rotating sphere. *Geophys.* Chandrasekhar, S. 1961 *Hydrodynamic and hydromagnetic stability*. Oxford: Clarendon.
Fearn, D. R. 1979 Thermal and magnetic instabilities in a rapidly rotating sphere. *Geophys.*
Astrophys. Fluid Dyn. **14**, 103–126. Fearn, D. R. 1979 Thermal and magnetic instabilities in a rapidly rotating sphere. *Geophys.*
Astrophys. Fluid Dyn. 14, 103–126.
Fearn, D. R. 1997 The geodynamo. In *Earth's deep interior* (ed. D. Crossley), pp. 79–114.
Lo
- Astrophys. Fluid Dyn. 14, 10
arn, D. R. 1997 The geodyn
London: Gordon & Breach.
arn, D. B. 1998 Hydromagnet Fearn, D. R. 1997 The geodynamo. In *Earth's deep interior* (ed. D. Crossley), pp. 79-
London: Gordon & Breach.
Fearn, D. R. 1998 Hydromagnetic flow in planetary cores. *Rep. Prog. Phys* 61, 175–235.
Fearn D. B. & Proster,
-
- Fearn,D. R. 1998 Hydromagnetic flow in planetary cores. *Rep. Prog. Phys* 61, 175–235.
Fearn, D. R. & Proctor, M. R. E. 1983 Hydromagnetic waves in a differentially rotating sphere.
J. Fluid Mech 128, 1–20 Fearn, D. R. & Proctor, M. R. E. 1983 Hydromagnetic waves in a differentially rotating sphere.
J. Fluid Mech. **128**, 1–20.
- Fearn,D.R.&Proctor,M. R. E. 1987 On the computation of steady, self-consistent spherical dynamos. *Geophys. Astrophys. Fluid Dyn.* 38, 293-325. Fearn, D. K. & Proctor, M. K. E. 1987 On the computation of steady, self-consistent spherical
dynamos. *Geophys. Astrophys. Fluid Dyn.* **38**, 293–325.
Glatzmaier, G. A. & Roberts, P. H. 1995a A three-dimensional convectiv
- dynamos. *Geophys. Astrophys. Fluid Dyn.* **38**, 293–325.
atzmaier, G. A. & Roberts, P. H. 1995a A three-dimensional convective dynamo solution
with rotating and finitely conducting inner core and mantle. *Phys. Earth Plane* with rotating and finitely conducting inner core and mantle. *Phys. Earth Planet. Interiors* **91**, 63–75. withrotating and finitely conducting inner core and mantle. *Phys. Earth Planet. Interiors* $91, 63-75$.
Glatzmaier, G. A. & Roberts, P. H. 1995b A three-dimensional self-consistent computer simulation of a geomagnetic f
- 91, 63–75.
atzmaier, G. A. & Roberts, P. H. 1995b A three-dimensional
lation of a geomagnetic field reversal. *Nature* 377, 203–208.
atzmaier, C. A. & Boberts, P. H. 1996s, Botation and magne Glatzmaier,G. A. & Roberts, P. H. 19950 A three-dimensional self-consistent computer simulation of a geomagnetic field reversal. *Nature* 377, 203–208.
Glatzmaier, G. A. & Roberts, P. H. 1996a Rotation and magnetism of th
- *A* a geomagnetic he
 Science 274, 1887–1891.
 Science 274, 1887–1891.
- Glatzmaier,G. A. & Roberts, P. H. 1996a Rotation and magnetism of the Earth's inner core.
 Science 274, 1887–1891.

Glatzmaier, G. A. & Roberts, P. H. 1996b An an[elastic evolutionary g](http://pippo.ingentaselect.com/nw=1/rpsv/cgi-bin/linker?ext=a&reqidx=/0167-2789^28^2997L.81[aid=539430,doi=10.1016/0031-9201^2894^2902965-E])eodynamo simulation

driven by comp deriven by compositional and thermal convection. *Physica* D97, 81–94.
driven by compositional and thermal convection. *Physica* D97, 81–94.
white D, 1977 Eperwatics of the Ferth's server L Cosphus, 43, 452, 464. driven by compositional and thermal convection. *Physica* D 97, 81–94.
Gubbins, D. 1977 Energetics of the Earth's core. *J. Geophys.* 43, 453–464.
- THE ROYAL
- driven by compositional and thermal convection. *Physica* D 97, 81–94.
Gubbins, D. 1977 Energetics of the Earth's core. *J. Geophys.* 43, 453–464.
Gubbins, D. 1999 The distinction between geomagnetic excursions and reversa *Ibbins, D. 1977 Ene*
Int. **137**, F1–F3.
Int. **137**, F1–F3. Gubbins, D. 1999 The distinction between geomagnetic excursions and reversals. Geophys. J.
 Int. **137**, F1–F3.

Gubbins, D. & Roberts, P. H. 1987 Magnetohydrodynamics of the Earth's core. In *Geomagnetism*

(ed. I. A. Ja
	- *Int.* **13**7, F1–F3.
(bbins, D. & Roberts, P. H. 1987 Magnetohydrodynamics)
(ed. J. A. Jacobs), vol. II, ch. 1, pp. 1–183. Academic.
(bbins, D. & Zhang, K. 1992. Summetry properties of t Gubbins, D. & Koberts, P. H. 1987 Magnetohydrodynamics of the Earth's core. In *Geomagnetism*
(ed. J. A. Jacobs), vol. II, ch. 1, pp. 1–183. Academic.
Gubbins, D. & Zhang, K. 1993 Symmetry properties of the dynamo equation
	- (ed. J. A. Jacobs), vol. II, ch. 1, pp. 1–183. Academic.
ibbins, D. & Zhang, K. 1993 Symmetry properties of the dynamo equa
netism and geomagnetism. *Phys. Earth Planet. Interiors* 75, 225–241.
ibbins, D. Mesters, T. C. ⁶ netismand geomagnetism. *Phys. Earth Planet. Interiors* 75, 225–241.
Gubbins, D., Masters, T. G. & Jacobs, J. A. 1979 Thermal evolution of the Earth's core. *Geophys.*
	- *J. R. Astr. Soc.* 59, 57-99.
	- Gubbins, D., Barber, C. N., Gibbons, S. & Love, J. J. 2000^a Kinematic dynamo action in J. R. Astr. Soc. 59, 57–99.
ibbins, D., Barber, C. N., Gibbons, S. & Love, J. J. 2000a Kinematic dynamo action in
a sphere. I. Effects of differential rotation and meridional circulation. *Proc. R. Soc. Lond.*
A 456 (In th bbins, D., Barber, C. I
a sphere. I. Effects of d
A 456. (In the press.) a sphere. I. Effects of differential rotation and meridional circulation. *Proc. R. Soc. Lond.*
A 456. (In the press.)
Gubbins, D., Barber, C. N., Gibbons, S. & Love, J. J. 2000b Kinematic dynamo action in a
sphere II Sym
	- A 456. (In the press.)
ibbins, D., Barber, C. N., Gibbons, S. & Love, J. J. 2000b Kinematic dyna
sphere. II. Symmetry selection. *Proc. R. Soc. Lond.* A 456. (In the press.)
mitted McKangia D. R. & Weiss, N. O. 1975 Discip Gubbins, D., Barber, C. N., Gibbons, S. & Love, J. J. 2000b Kinematic dynamo action in a sphere. II. Symmetry selection. *Proc. R. Soc. Lond.* A 456. (In the press.)
Hewitt, J., McKenzie, D. P. & Weiss, N. O. 1975 Dissipat
	- *sphere. II. Symmetry*
 witt, J., McKenzie, D
 Mech. **68**, 721–738. *Phil. Trans. R. Soc. Lond.* A (2000)

IATHEMATICAL,
HYSICAL
ENGINEERING
CIENCES

Downloaded from rsta.royalsocietypublishing.org

Scaledisparitiesinthegeodynamo ⁹¹⁹

- Hollerbach, R. 1996 On the theory of the geodynamo. *Phys. Earth Planet. Interiors* **98**, 163–185.
Hollerbach, R. 1997 The dynamical balance in semi-Taylor states. *Geophys. Astrophys. Fluid*
Dun 84, 85–98 ICAL
GINEERING
VCES Hollerbach, R. 1996 On the theory of the geodynamo. *[Phys. Earth Planet. Interiors](http://pippo.ingentaselect.com/nw=1/rpsv/cgi-bin/linker?ext=a&reqidx=/0031-9201^28^2998L.163[aid=531348,doi=10.1016/0031-9201^2895^2903068-8])* 98, 163–185. *Derbach, R. 1996*
Dyn. 84, 85–98.
Dyn. 84, 85–98.
	- Hollerbach, R. 1997 I he dynamical balance in semi-Taylor states. Geophys. Astrophys. Fund
Dyn. 84, 85–98.
Hollerbach, R. & Jones, C. A. 1993 Influence of the Earth's inner core on geomagnetic fluctua-
tions and reversals. *tyn.* 84, 85–98.
Illerbach, R. & Jones, C. A. 1993 Influence of
tions and reversals. *Nature* 365, 541–543.
Illerbach, B. & Jones, C. A. 1995 On the r
	- tions and reversals. *Nature* **365**, 541–543.
Hollerbach, R. & Jones, C. A. 1995 On the magnetically stabilizing role of the Earth's inner core. *Phys. Earth Planet. Interiors* 87, 171–181.
	- Hollerbach,K. & Jones, C. A. 1995 On the magnetically stabilizing role of the Earth's inner
core. *Phys. Earth Planet. Interiors* 87, 171–181.
Hutcheson, K. A. & Gubbins, D. 1994 Kinematic magnetic field morphology at th core. *Phys. Earth Planet. Interiors* 87, 171⁻¹
itcheson, K. A. & Gubbins, D. 1994 Kinema
boundary. *Geophys. J. Int.* **116**, 304–320.
ult. D. 1005 Model Z computation and Tay Hutcheson, K. A. & Gubbins, D. 1994 Kinematic magnetic field morphology at the core mantle
boundary. *Geophys. J. Int.* **116**, 304–320.
Jault, D. 1995 Model Z computation and Taylor's condition. *Geophys. Astrophys. Fluid*
	- 60 boundary. Get

	1995 1
 79, 99–124.
	- Jault, D. 1995 Model Z computation and Taylor's condition. Geophys. Astrophys. Fiuld Dyn.

	79, 99–124.

	Jones, C. A., Longbottom, A. W. & Hollerbach, R. 19[95 A self-consistent convection driven](http://pippo.ingentaselect.com/nw=1/rpsv/cgi-bin/linker?ext=a&reqidx=/0031-9201^28^2992L.119[aid=539394,doi=10.1016/S0031-9201^2896^2903207-4])

	recodynamo model using a me 79, 99–124.
nes, C. A., Longbottom, A. W. & Hollerbach, R. 1995 A self-consistent convection driven
geodynamo model, using a mean field approximation. *Phys. Earth Planet. Interiors* 92, 119–
141 141.
	- geodynamo model, using a mean neid approximation. *Phys. Earth Pianet. Interiors* 92, 119–
141.
Jones, C. A., Soward, A. M. & Mussa, A. I. 2000 The onset of thermal convection in a rapidly
rotating sphere *L. Fluid Mech* 4 141.
nes, C. A., Soward, A. M. & Mussa, A. I. 2000
rotating sphere. *J. Fluid Mech.* 405, 157–179.
ttarame. I. S. Matauchima. I. S. M. & Healmy
	- Jones,C. A., Soward, A. M. & Mussa, A. I. 2000 The onset of thermal convection in a rapidly
rotating sphere. J. Fluid Mech. 405, 157–179.
Katayama, J. S., Matsushima, J. S. M. & Honkura, Y. 1999 Some characteristics of m rotating sphere. *J. Fund Mech.* 405, 157–179.
tayama, J. S., Matsushima, J. S. M. & Honkura, Y. 1999 Some characteristics of magnetic
field behavior in a model of MHD dynamo thermally driven in a rotating spherical shell. *Earth Planet.* I. S., Matsushima, J. S. M. &
 Earth Planet. Interiors 111, 141–159.
 Earth Planet. Interiors 111, 141–159. fieldbehavior in a model of MHD dynamo thermally driven in a rotating spherical shell. *Phys.*
Earth Planet. Interiors 111, 141–159.
Kuang, W. & Bloxham, J. 1997 An Earth-like numerical dynamo model. *Nature* 389, 371–3
	-
	- EarthPlanet. Interiors 111, 141–159.
Kuang, W. & Bloxham, J. 1997 An Earth-like numerical dynamo model. *Nature* 389, 371–374.
Kumar, S. & Roberts, P. H. 1975 A three-dimensional kinematic dynamo. *Proc. R. Soc. Lond.*
A ang, W. & Bloxna
umar, S. & Robert
A 344, 235–238.
brosse. S. Poirier. Kumar, S. & Roberts, P. H. 1975 A three-dimensional kinematic dynamo. *Proc. R. Soc. Lond.*
A 344, 235–238.
Labrosse, S., Poirier, J.-P. & LeMouël, J.-L. 1997 On cooling of the Earth's core. *[Phys. Earth](http://pippo.ingentaselect.com/nw=1/rpsv/cgi-bin/linker?ext=a&reqidx=/0031-9201^28^2999L.1[aid=531354,doi=10.1016/0031-9201^2895^2903065-5])*
Planet Interiors
	- *A* **344**, 235–238.

	brosse, S., Poirier, J.-P. & L.
 Planet. Interiors **99**, 1–17.
 p.g. D. E. 1978. Some thermes Labrosse,S., Poirier, J.-P. & LeMouël, J.-L. 1997 On cooling of the Earth's core. *Phys. Earth*
 Planet. Interiors 99, 1–17.

	Loper, D. E. 1978 Some thermal consequences of a gravitationally powered dynamo. *J. Geophys.*
	- *Planet. Interiors* **99**,

	per, D. E. 1978 Some t
 Res. **83**, 5961–5970.
 real legislation.
	- Res. 83, 5961–5970.
Love, J. & Gubbins, D. 1996 Dynamos driven by poloidal flows exist. *Geophys. Res. Lett.* 23, 857–860. Love, J. & Gubbins, D. 1996 Dynamos driven by poloidal flows exist. *Geophys. Res. Lett.* 23,
857–860.
Moffatt, H. K. 1978 *Magnetic field generation in electrically conducting fluids*. Cambridge Uni-
versity Press
	- 857–800.
offatt, H. K. 19'
versity Press. versity Press.
Olson, P. & Glatzmaier, G. 1995 Magnetoconvection in a rotating spherical shell: structure of
	- flow in the outer core. *Phys. Earth Planet. Interiors* 92, 109-118.
	- Olson, P. & Glatzmaier, G.1996Magnetoconvectionandthermalcoupling of the Earth' s core and mantle. *Phil. Trans. R. Soc. Lond.* A 354, 1-12. Olson, P., & Glatzmaier, G. 1996 Magnetoconvection and thermal coupling of the Earth's core
and mantle. *Phil. Trans. R. Soc. Lond.* A 354, 1–12.
Olson, P., Christensen, U. & Glatzmaier, G. 1999 Numerical modeling of the
	- and mantle. *Phil. Trans. R. Soc. Lond.* A 354, 1–12.
son, P., Christensen, U. & Glatzmaier, G. 1999 Numerical modeling of the geodynam
anisms of field generation and equilibration. *J. Geophys. Res.* 104, 10 383–10 404. anismsof field generation and equilibration. *J. Geophys. Res.* 104, 10383–10404.
Proctor, M. R. E. 1994 Magnetoconvection in a rapidly rotating sphere. In *Stellar and planetary*
	- *dynamos* (ed. M. R. E. Proctor & A. D. Gilbert). Cambridge University Press. Proctor, M. R. E. 1994 Magnetoconvection in a rapidly rotating sphere. In *Stellar and planetary*
dynamos (ed. M. R. E. Proctor & A. D. Gilbert). Cambridge University Press.
Roberts, P. H. 1965 On the thermal instability o
	- aynamos (ed. M
berts, P. H. 196
141, 240–250.
berts, P. H. 106
	- Roberts,P. H. 1968 On the thermal instability of a self-gravitating fluid sphere containing heat sources. *Phil. Trans. R. Soc. Lond.* A 263, 93–117. Roberts, P. H. 1968 On the thermal instability of a self-gravitating fluid sphere containing
sources. *Phil. Trans. R. Soc. Lond.* A 263, 93-117.
Roberts, P. H. & Soward, A. M. 1992 Dynamo theory. *A. Rev. Fluid Dyn.* 24,
	-
- Sources. *That. Trans. R. 50c. Long.* A 205, $35-111$.
Roberts, P. H. & Soward, A. M. 1992 Dynamo theory. A. Rev. Fluid Dyn. 24, 459–512.
Sarson, G. & Gubbins, D. 1996 Three-dimensional kinematic dynamos dominated by st berts, P. H. & Soward, A. M. 1992 Dynamo theory.
rson, G. & Gubbins, D. 1996 Three-dimensional ki
differential rotation. *J. Fluid Mech.* **306**, 223–265. Sarson,G. & Gubbins, D. 1996 Three-dimensional kinematic dynamos dominated by strong
differential rotation. *J. Fluid Mech.* **306**, 223–265.
Sarson, G. R. & Jones, C. A. 1999 A convection driven geodynamo reversal model.
	- *Planet.* Interiors*in. J. Fluid*
 Planet. Interiors 111, 3-20.
 Planet. Interiors 111, 3-20. Sarson,G. R., & Jones, C. A. 1999 A convection driven geodynamo reversal model. *Phys. Earth*
 Planet. Interiors 111, 3–20.

	Sarson, G. R., Jones, C. A. & Longbottom, A. W. 1998 Convection driven geodynamo models

	of va
	- Planet. Interiors 111, 3–20.
rson, G. R., Jones, C. A. & Longbottom, A. W. 1998 Convection driven go
of varying Ekman number. *Geophys. Astrophys. Fluid Dyn.* **88**, 225–259. *Phil. Trans. R. Soc. Lond.* A (2000) *Phil. Trans. R. Soc. Lond.* A (2000)

IATHEMATICAL,
HYSICAL
ENGINEERING **THEERING**

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PHILOSOPHICAL
TRANSACTIONS

ATHEMATICAL

- 920 $\,$ K. Zhang and D. Gubbins
Soward, A. M. 1977 On the finite amplitude thermal instability of a rapidly rotating fluid sphere. *Geophys. Astrophys. Fluid Dyn.* 9, 19-74.
- **MATHEMATICAL,
PHYSICAL**
& ENGINEERING
SCIENCES Soward, A. M. 1977 On the finite amplitude thermal instability of a rapidly rotating fluid sphere.
 Geophys. Astrophys. Fluid Dyn. 9, 19–74.

Taylor, J. B. 1963 The magneto-hydrodynamics of a rotating fluid and the eart *Geophys. Astrophys. Fluid Dyn.* **9**, 19–74.

ylor, J. B. 1963 The magneto-hydrodynamics

problem. *Proc. R. Soc. Lond.* A 274, 274–283.

Nex. M. B. & Hollerbach. B. 1999 The adjustm problem. Proc. R. Soc. Lond. A 274, 274–283.
Walker, M. R. & Hollerbach, R. 1999 The adjustment to Taylor's constraint in the presence of
	- an ambient field. *Phys. Earth Planet. Interiors* 114, 181-196.
	- Walker,M. R. & Hollerbach, R. 1999 The adjustment to Taylor's constraint in the presence of
an ambient field. *Phys. Earth Planet. Interiors* 114, 181–196.
Walker, M. R., Barenghi, C. F. & Jones, C. A. 1998 A note on dyna an ambient field. *Phys. Earth Planet. Interiors* 114, 181–196.
alker, M. R., Barenghi, C. F. & Jones, C. A. 1998 A note on dynamo ao
small Ekman number. *Geophys. Astrophys. Fluid Dyn.* 88, 261–275.
ang K. 1991 Convection
	- Walker, M. K., Barenghi, C. F. & Jones, C. A. 1998 A note on dynamo action at asymptotically
small Ekman number. *Geophys. Astrophys. Fluid Dyn.* 88, 261–275.
Zhang, K. 1991 Convection in a rapidly rotating spherical fluid small Ekman number. *Geophys. Astrophys. Fluid Dyn.* 88, 261–275 ang, K. 1991 Convection in a rapidly rotating spherical fluid shell at steadily drifting rolls. *Phys. Earth Planet. Interiors* 68, 156–169. Zhang,K. 1991 Convection in a rapidly rotating spherical fluid shell at infinite Prandtl number:
steadily drifting rolls. *Phys. Earth Planet. Interiors* 68, 156–169.
Zhang, K. 1992 Spiralling columnar convection in rapid
	- *Mech.* 236, 535–556.
Mech. 236, 535–556.
Mech. 236, 535–556. Zhang,K. 1992 Spiralling columnar convection in rapidly rotating spherical fluid shells. *J. Fluid*
 Mech. 236, 535–556.

	Zhang, K. 1994 On coupling between the Poincaré equation and the heat equation. *J. Fluid*
 Mech
	- *Mech.* **236**, 535–556.

	ang, K. 1994 On coup
 Mech. **268**, 211–229.

	ang K. 1995*e* Spherics Zhang, K. 1994 On coupling between the Poincare equation and the heat equation. J. Fluid Mech. 268, 211–229.
Zhang, K. 1995a Spherical shell rotating convection in the presence of a toroidal magnetic field.
Proc. B. Soc.
	- *Mech.* **268**, 211–229.

	ang, K. 1995a Spherical shell rotating *Proc. R. Soc. Lond.* A 448, 245–268.

	ang, K. 1995b On coupling between the
	- Zhang, K. 1995a Spherical shell rotating convection in the presence of a toroidal magnetic field.
 Proc. R. Soc. Lond. A 448, 245–268.

	Zhang, K. 1995b On coupling between the Poincaré equation and the heat equation: no Proc. R. Soc. Lond. A 448, 245–268.
ang, K. 1995b On coupling between the Poincaré ee
boundary condition. *J. Fluid Mech.* 284, 239–256.
ang. K. 1999 Nonlinear magnetobydrodynamic.com Zhang, K. 1995b On coupling between the Poincaré equation and the heat equation: non-slip
boundary condition. J. Fluid Mech. 284, 239–256.
Zhang, K. 1999 Nonlinear magnetohydrodynamic convective flows in the Earth's fluid
	- *Phys. Earth Planet. J. Fluid Mech.* 284, 239
Phys. Earth Planet. Interiors 111, 93–105.
 Phys. Earth Planet. Interiors 111, 93–105. Zhang,K. 1999 Nonlinear magnetohydrodynamic convective flows in the Earth's fluid core.
 Phys. Earth Planet. Interiors 111, 93–105.

	Zhang, K. & Gubbins, D. 2000 Is the geodynamo process intrinsically unstable? *Geophys*
	- *Phys. Earth Plane:*
ang, K. & Gubbin
Int. **140**, F1–F4. Zhang, K. & Gubbins, D. 2000 is the geodynamo process intrinsically unstable? Geophys. J.

	Int. 140, F1–F4.

	Zhang, K. & Jones, C. A. 1993 The influence of Ekman boundary layers on rotating convection.

	Geophys Astrophys
	- *Int.* **140**, F1–F4.
ang, K. & Jones, C. A. 1993 The influence of El
Geophys. Astrophys. Fluid Dyn. **71**, 145–162.
ang K. & Jones C. A. 1994 Convective metions Zhang, K. & Jones, C. A. 1993 The influence of Ekman boundary layers on rotating convection.
 Geophys. Astrophys. Fluid Dyn. **71**, 145–162.

	Zhang, K. & Jones, C. A. 1994 Convective motions in the Earth's fluid core. *Ge*
	- Geophys. Astropl
ang, K. & Jones,
21, 1939–1942. Zhang, K. & Jones, C. A. 1994 Convective motions in the Earth's fluid core. *[Geophys.](http://pippo.ingentaselect.com/nw=1/rpsv/cgi-bin/linker?ext=a&reqidx=/0094-8276^28^2924L.2869[aid=539423,doi=10.1016/0031-9201^2895^2903049-3]) Res. Lett.*
21, 1939–1942.
Zhang, K. & Jones, C. A. 1997 The effect of hyperviscosity on geodynamo models. *Geophys.*
Res. Lett. 24, 286
	- **21**, 1939–1942.

	ang, K. & Jones, C. A. 19:
 Res. Lett. **24**, 2869–2872.

	ang, K. K. & Busso, E. H. Zhang,K.-K. Jones, C.-A. 1997 The effect of hyperviscosity on geodynamo models. *Geophys.*
 Res. Lett. 24, 2869–2872.

	Zhang, K.-K. & Busse, F.-H. 1998 Some recent developments in the theory of convection in

	rotating
	- *Res. Lett.* **24**, 2869–2872.
ang, K.-K. & Busse, F. H. 1998 Some recent c
rotating systems. *Adv. Fluid Mech.* **20**, 17–70.

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